

Math 54 Discussion Section Problems

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February 20, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

- Use Cramer's rule to solve
$$\begin{cases} 3x - 2y + 5z = 14 \\ -x + 4y - z = 4 \\ x + z = 4 \end{cases}$$
- Suppose A is a 3×3 matrix with determinant 5.
 - What is the reduced echelon form of A ?
 - What are $\det(2A)$, $\det A^{-1}$, $\det A^T$, $\det A^3$?
- Let S be the parallelogram determined by the vectors $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$, and let $A = \begin{bmatrix} 6 & -2 \\ -3 & 2 \end{bmatrix}$
 - Sketch S . Use determinants to find its area.
 - Now find the area of the image of S under $\mathbf{x} \mapsto A\mathbf{x}$.
- It turns out that we can also use determinants to find the area/volume of things that aren't simple parallelograms/parallelepipeds. Here's an example in which we calculate the volume of an ellipsoid (the 3d version of an ellipse).
 - Let S be the unit ball (ie, a sphere of radius 1 centered at the origin) and let T be the map $\mathbf{x} \mapsto \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \mathbf{x}$. Sketch $T(S)$. Can you find an equation for it? Hint: an equation for the sphere of radius 1 is $x^2 + y^2 + z^2 = 1$
 - Find the volume of $T(S)$
 - For those who have taken math 53: notice how this is much easier than trying to take some awful triple integral, probably in spherical coordinates.
- Prove that if A and B are square matrices such that AB is invertible, then both A and B must be invertible.