

Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Let $A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & 3 & -9 \end{pmatrix}$.

Find:

- (a) B^{-1}
- (b) AB^{-1}
- (c) $(B^T)^{-1}A^T$

2. Is it possible to have a 4×3 invertible matrix? How about a 3×4 ? Why or why not?

3. Let $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{pmatrix}$. Find a 2×3 matrix C such that $CA = I_2$. Note: you can do this using only 0, 1, -1 as entries. Now compute AC . Do you get the identity? What does this mean about the invertibility of A ?

4. True/false. For those that are true, explain why. For those that are false, give a counterexample. For all of the below problems, A is a square matrix.

- (a) If $A\mathbf{x} = \mathbf{b}$ has a solution for every \mathbf{b} , then A is invertible.
- (b) If $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution, then A is invertible.
- (c) If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then the columns of A span \mathbb{R}^n .
- (d) There is a matrix A such that A is invertible but A^2 is not.
- (e) If AB is invertible, then A is invertible.