

Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Find the general solution to the system of equations whose augmented matrix is

(a) $\begin{pmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{pmatrix}$

2. What is $\begin{pmatrix} 2 & -1 & 1 \\ -6 & 3 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$?

3. Write the system of equations $\begin{cases} 4x_1 - 3x_2 = 4 \\ x_1 + 4x_2 = 1 \\ -x_1 + 3x_2 = 7 \end{cases}$ as a matrix equation.

4. Let $A = \begin{pmatrix} 2 & -1 \\ -6 & 3 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$. For what values of b_1, b_2 does the equation $A\mathbf{x} = \mathbf{b}$ have a solution? What does this mean in terms of the span of the columns of A ?

5. Let $A = \begin{pmatrix} 2 & -1 \\ -6 & a \end{pmatrix}$. For what values of a does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for **every** vector \mathbf{b} ? Phrased another way, for what values of a do the columns of A span \mathbb{R}^2 ?

6. As you may (or may not) have learned at some point, there is only one polynomial of degree n that passes through a given set of $n + 1$ points (ie, there is only one line through any given 2 points, only one parabola through a given 3 points, etc). What you may not have learned, however, is that linear algebra can be used to find this polynomial.

- (a) Set up and solve a system of 3 equations in 3 unknowns that will let you find the parabola that passes through the points $(1, 12), (2, 15), (3, 16)$. Hint: a parabola has equation $p(t) = x_1 + x_2t + x_3t^2$, so you just need to find the correct x_1, x_2, x_3 that make $p(1) = 12$, etc.
- (b) Using what we've already learned about systems of linear equations, determine how many parabolas pass through the points $(1, 12)$ and $(2, 15)$.