

Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Determine whether or not each of the following equations is **linear**.

(a) $x_1 + 3x_2 = -\pi$

(b) $x_1 + x_1x_2 = 4$

(c) $\sin(x) + x = 14$

2. For each of the below systems of equations, (i) find the Reduced Echelon form of the augmented matrix for the system of equations and (ii) determine whether the system has no solutions, exactly 1 solution, or infinitely many solutions. For those that have exactly 1 solution, find it.

(a)

$$\begin{aligned}x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5\end{aligned}$$

(b)

$$\begin{aligned}x_1 + 3x_2 + 5x_3 &= -2 \\ x_2 + 4x_3 &= -5 \\ 3x_1 + 7x_2 + 7x_3 &= 6\end{aligned}$$

(c)

$$\begin{aligned}x_1 + 3x_2 + 6x_3 &= 8 \\ x_1 + 3x_2 + 5x_3 &= 7 \\ 2x_1 + 6x_2 + 13x_3 &= 17\end{aligned}$$

(d)

$$\begin{aligned}4x_1 + 8x_2 - 4x_3 &= 7 \\ -2x_1 + 18x_2 + 2x_3 &= 0 \\ -3x_1 + 5x_2 + 3x_3 &= -2\end{aligned}$$

3. Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ all intersect in a single point?

4. Despite what the above examples may lead you to believe, there is no reason why a system of equations in n variables has to have n equations. For each of the following situations, either come up with an example that demonstrates it, or explain why no such example can exist:

(a) A system of 3 equations in 2 unknowns with no solutions

(b) A system of 3 equations in 2 unknowns with exactly 1 solution

(c) A system of 3 equations in 2 unknowns with infinitely many solutions

(d) A system of 2 equations in 3 unknowns with no solutions

(e) A system of 2 equations in 3 unknowns with exactly 1 solution

(f) A system of 2 equations in 3 unknowns with infinitely many solutions