Name: ____________________
Section: 9-10 11-12 2-3

Math 54 Quiz 7 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

1. (2pts) Define each of the following:

   (a) Eigenvalue and a corresponding eigenvector

   λ is an eigenvalue with corresponding eigenvector $\mathbf{v}$ for a matrix A iff $\mathbf{v} \neq 0$ and $A\mathbf{v} = \lambda\mathbf{v}$

   (b) Diagonalizable

   A is diagonalizable iff A is similar to a diagonal matrix.

2. (4pts) Find the eigenvalues of $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 3 \\ 3 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 9 = \lambda^2 - 2\lambda - 8 = (\lambda-4)(\lambda+2)$$

So the eigenvalues are $4, -2$

3. (4pts) Let $A = \begin{bmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

   (a) Find the eigenvalues and bases for the corresponding eigenspaces of A.

   This is upper triangular, so the eigenvalues are just the entries on the diagonal. Namely, $5, 3, 2$.

   $(\lambda = 5):$

$$\begin{bmatrix} 0 & -3 & 0 & 9 \\ 0 & -2 & 1 & -2 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & -3 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   So we have x_1 is free and $x_2 = x_3 = x_4 = 0$. So a basis for the eigenspace corresponding to $\lambda = 5$ is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

   $(\lambda = 3):$

$$\begin{bmatrix} 2 & -3 & 0 & 9 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So $x_3 = x_4 = 0$, x_2 is free and $x_1 = \frac{3}{2}x_2$. So a basis is $\begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$(\lambda = 2)$:

$$\begin{bmatrix} 3 & -3 & 0 & 9 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 3 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

And these two vectors form a basis for the eigenspace corresponding to $\lambda = 2$

(b) Determine if A is diagonalizable. If yes, diagonalize it. If no, explain how you know.

We have 4 linearly independent eigenvectors in $\mathbb{R}^4$, so it is diagonalizable. $D = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, P = \begin{bmatrix} 1 & 3/2 & -1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$