Name: ______________________

Section: 9-10 11-12 2-3

Math 54 Quiz 6 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

1. (4 pts) Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \end{bmatrix} \right\}, \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$

   (a) Find $\mathcal{P}_{\mathcal{C}\leftarrow\mathcal{B}}$

$$\left[\begin{array}{cc|cc} 1 & -2 & 7 & -3 \\ -5 & 2 & 5 & -1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -2 & 7 & -3 \\ 0 & -8 & 40 & -16 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -2 & 7 & -3 \\ 0 & 1 & -5 & 2 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & -3 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

   So $\mathcal{P}_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} -3 & 1 \\ -5 & 2 \end{bmatrix}$

   (b) Find $\mathcal{P}_{\mathcal{B}\leftarrow\mathcal{C}}$

   $\mathcal{P}_{\mathcal{B}\leftarrow\mathcal{C}} = \mathcal{P}_{\mathcal{C}\leftarrow\mathcal{B}}^{-1} = \frac{1}{-6+5} \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -5 & 3 \end{bmatrix}$

2. (3 pts) Suppose A is a 10×14 matrix, $\mathbf{c} \in \mathbb{R}^{10}$ and that the solution set to $A\mathbf{x} = \mathbf{c}$ has 4 free variables. Prove that the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto.

   If $Ax = c$ has 4 free variables in its solution, then dim Nul $A = 4$, so dim Col $A = 14 - 4 = 10$. So Col A is a 10-dimensional subspace of $\mathbb{R}^{10}$ and thus must be all of $\mathbb{R}^{10}$. Thus, $x \mapsto Ax$ is onto since every vector in $\mathbb{R}^{10}$ is a linear combination of the columns of A.

3. (3 pts) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for a vector space V and suppose $\mathbf{b}_1 = 3\mathbf{c}_1 + 4\mathbf{c}_2, \mathbf{b}_2 = 2\mathbf{c}_1 - \mathbf{c}_2$. Find the change of coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$

   $\mathcal{P}_{\mathcal{C}\leftarrow\mathcal{B}} = [[\mathbf{b}_1]_{\mathcal{C}}\, [\mathbf{b}_2]_{\mathcal{C}}] = \begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}$