Name: ______________________
Section: 9-10 11-12 2-3

Math 54 Quiz 3 SOLUTIONS

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You have twenty minutes to complete this quiz. You must show your work.

1. (4 pts) Let $A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -1 & 1 \\ 0 & 1 \end{pmatrix}$

   (a) Find A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 1 & 0 & 0 \\ 0 & 3 & -2 & 0 & 1 & 0 \\ -2 & 6 & 3 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 1 & 0 & 0 \\ 0 & 3 & -2 & 0 & 1 & 0 \\ 0 & 6 & -5 & 2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 1 & 0 & 0 \\ 0 & 3 & -2 & 0 & 1 & 0 \\ 0 & 0 & -1 & 2 & -2 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7 & 8 & -4 \\ 0 & 3 & 0 & -4 & 5 & -2 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{array}\right]$$

   So $A^{-1} = \begin{bmatrix} -7 & 8 & -4 \\ -4/3 & 5/3 & -2/3 \\ -2 & 2 & -1 \end{bmatrix}$

   (b) Compute $B^T(3A^{-1})$ Note: if you are in the 9am section, your test didn't have the 3 in it–your answer should be 1/3 of the one below:

$$\begin{aligned} B^T(3A^{-1}) &= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -21 & 24 & -12 \\ -4 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -21+4 & 24-5 & -12+2 \\ 42-4-6 & -48+5+6 & 24-2-3 \end{bmatrix} = \begin{bmatrix} -17 & 19 & -10 \\ 32 & -37 & 19 \end{bmatrix} \end{aligned}$$

2. (3 pts) For what value(s) of h are $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix}, \begin{pmatrix} -1 \\ 5 \\ h \end{pmatrix}$ linearly **dependent**?

   They are linearly dependent if $x_1 \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 5 \\ h \end{pmatrix} = 0$ has a non-trivial solution. In matrix form, we are looking for a non-trivial solution to:

$$\begin{bmatrix} 1 & 3 & -1 \\ -1 & -5 & 5 \\ 4 & 7 & h \end{bmatrix} \mathbf{x} = \mathbf{0}$$

We do row reductions:

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ -1 & -5 & 5 & 0 \\ 4 & 7 & h & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & -5 & h+4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -5 & h+4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & h-6 & 0 \end{array}\right]$$

This will have non-trivial solutions if and only if there is a non-pivot column (ie, a free variable). We can already see that columns 1 and 2 are pivot columns no matter what we choose for h, so our only hope for a non-pivot is if $h - 6 = 0$. That is to say, we need $h = 6$

3. (3 pts) Write down 6 different conditions that are equivalent to an $n \times n$ matrix A being invertible:

   (a) A has n pivots

   (b) $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$

   (c) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution

   (d) The columns of A are linearly independent

   (e) The columns of A span $\mathbb{R}^n$

   (f) There is a matrix C such that $AC = I_n$

   (g) There is a matrix D such that $DA = I_n$

   (h) The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is 1-1

   (i) The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto (this is really the same as (b))

   (j) $\det A \neq 0$

   (k) See book for more