Name: ____________________

Section: 9-10 11-12 2-3

Math 54 Quiz 2

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You have twenty minutes to complete this quiz. You must show your work.

1. (4 pts) Find the general solution to the system of equations $\begin{cases} x_1 + 3x_2 - 5x_3 = 4 \\ x_1 + 4x_2 - 8x_3 = 7 \\ -3x_1 - 7x_2 + 9x_3 = -6 \end{cases}$

   Write your answer in parametric vector form.

   We create the augmented matrix $\begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix}$

   Row operations will eventually give $\begin{bmatrix} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   So the solution is $x_1 = -5 - 4x_3, x_2 = 3 + 3x_3, x_3$ is free. Written in vector form, this is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

2. Let $\mathbf{u} = \begin{pmatrix} 4 \\ 1 \\ -4 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{pmatrix}$

   (a) (2 pts) Is $\mathbf{u}$ in the span of the columns of A?

   Asking if u is in the span of the columns of A is the same as asking if $A\mathbf{x} = \mathbf{u}$ has a solution, so we create an augmented matrix and check for consistency:

$$\begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

   So the system is consistent and thus $\mathbf{u}$ is in the span of the columns of A.

   (b) (4 pts) For which vectors $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is $\mathbf{b}$ in the span of the columns of A?

   This is the same as asking when $Ax = b$ has a solution. We do the same steps as above:

$$\begin{bmatrix} 1 & 0 & -4 & b_1 \\ 0 & 3 & -2 & b_2 \\ -2 & 6 & 3 & b_3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -4 & b_1 \\ 0 & 3 & -2 & b_2 \\ 0 & 6 & -5 & b_3 + 2b_1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -4 & b_2 \\ 0 & 3 & -2 & b_2 \\ 0 & 0 & -1 & b_3 + 2b_1 - 2b_2 \end{bmatrix}$$

Since there is a pivot in every row, this system will **always** be consistent, no matter what we choose for b_1, b_2, b_3. Thus, all vectors are in the span of the columns of A.