Name: ______________________

Section: 9-10 11-12 2-3

Math 54 Quiz 12 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

1. (3 pts) Using symbols (not pictures/descriptions), define what it means for a function $f(x)$ to be
   (a) Odd
   $f(-x) = -f(x)$
   (b) Even
   $f(-x) = f(x)$
   (c) $2T$ periodic
   $f(x+2T) = f(x)$

2. (3 pts) Write down the formulas for the Fourier series of f on the interval $(-T, T)$:

$$\begin{aligned} f &\sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{T} + b_n \sin\frac{n\pi x}{T} \\ a_n &= \frac{1}{T}\int_{-T}^{T} f(x)\cos\frac{n\pi x}{T}dx \\ b_n &= \frac{1}{T}\int_{-T}^{T} f(x)\sin\frac{n\pi x}{T}dx \end{aligned}$$

3. (4 pts) Find a formal solution to the vibrating string problem with

$$\begin{aligned} \alpha &= 3 \\ L &= \pi \\ u(x,0) &= 6\sin 2x + 2\sin 6x \\ u_t(x,0) &= 11\sin 9x - 14\sin 15x \end{aligned}$$

You may use the fact that the general solution to vibrating string problem is

$$u(x,t) = \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi\alpha}{L}t + b_n \sin\frac{n\pi\alpha}{L}t\right)\sin\frac{n\pi}{L}x$$

The general solution for this particular problem is $u(x,t) = \sum_{n=1}^{\infty}(a_n\cos 3nt + b_n \sin 3nt)\sin nx$

So $u(x,0) = \sum a_n \sin nx$, which we want to be equal to $6\sin 2x + 2\sin 6x$. So we will use $a_2 = 6, a_6 = 2$, and all others 0.

$$u_t(x,t) = \sum_{n=1}^{\infty}(-3na_n \sin 3nt + 3nb_n \cos 3nt)\sin nx$$

, so $u_t(x,0) = \sum 3nb_n \sin nx$

We want this to be $11\sin 9x - 14\sin 15x$, so we need $3\cdot 9b_9 = 11, 3\cdot 15b_{15} = -14$. This gives $b_9 = \frac{11}{27}, b_{15} = -\frac{14}{45}$.

Plugging these back in to our original general solution gives

$$u(x,t) = 6\cos 6t\sin 2x + 2\cos 18t \sin 6x + \frac{11}{27}\sin 27t\sin 9x - \frac{14}{45}\sin 45t \sin 15x$$