

Math 54 Quiz 11 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

1. Let L be the operator defined by $L := (D^2 + 1)(D + 2)^2(D^2 - 2D + 5)D^3$.
 Find the general solution to $L[y] = 0$

The characteristic polynomial is $(r^2 + 1)(r + 2)^2(r^2 - 2r + 5)r^3$,
 which has roots $\pm i, -2$ (mult. 2), $1 \pm 2i, 0$ (mult 3).

Thus, the general solution is

$$C_1 \cos t + C_2 \sin t + C_3 e^{-2t} + C_4 t e^{-2t} + C_5 e^t \cos 2t + C_6 e^t \sin 2t + C_7 + C_8 t + C_9 t^2$$

2. Rewrite the system of equations $\begin{cases} y'' + 3ty' + 6x' - 4y & = \cos t \\ x'' - e^t y' + x' + t^2 y - x & = \tan t \end{cases}$
 as a system of first order equations in matrix normal form.

Let's say $y_0 = y, y_1 = y', y_2 = x, y_3 = x'$

$$\begin{array}{l} \text{Then} \\ \begin{array}{l} y'_0 = y' = y_1 \\ y'_1 = y'' = 4y_0 - 3ty_1 - 6y_3 + \cos t \\ y'_2 = x' = y_3 \\ y'_3 = x'' = -t^2 y_0 + e^t y_1 + y_2 - y_3 + \tan t \end{array} \end{array}$$

$$\text{In matrix form, this is } \mathbf{y}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 4 & -3t & 0 & -6 \\ 0 & 0 & 0 & 1 \\ -t^2 & e^t & 1 & -1 \end{bmatrix} \mathbf{y} + \begin{bmatrix} 0 \\ \cos t \\ 0 \\ \tan t \end{bmatrix}$$

3. Let $L[y] := y''' - 3y'' + 4y$. Find the general solution to $L[y] = 20e^{3t} - 14 \cos t - 2 \sin t$.
 You may use the fact that $L[e^{3t}] = 4e^{3t}$ and $L[\cos t] = 7 \cos t + \sin t$

We really want to solve $y''' - 3y'' + 4y = 20e^{3t} - 14 \cos t - 2 \sin t$.

We first solve the homogeneous equation by solving $r^3 - 3r^2 + 4 = 0$. Guess and check tells us that $r = -1$ is a root and that this factors as $(r + 1)(r - 2)^2$, so the homogenous solution is $y_h = C_1 e^{-t} + C_2 e^{2t} + C_3 t e^{2t}$

We now want to find a y_p such that $L[y_p] = 20e^{3t} - 14 \cos t - 2 \sin t$. Based on what we're given, we see that $L[5e^{3t} - 2 \cos t] = 5L[e^{3t}] - 2L[\cos t] = 20e^{3t} - 14 \cos t - 2 \sin t$, so we'll use $y_p = 5e^{3t} - 2 \cos t$

The general solution is thus $y = y_h + y_p = C_1 e^{-t} + C_2 e^{2t} + C_3 t e^{2t} + 5e^{3t} - 2 \cos t$