Name: ______________________________
Section: 9-10 11-12 2-3

Math 54 Quiz 10 SOLUTIONS
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You have 20 minutes to complete this quiz. You must show your work.

1. (a) (1 pt) Complete the following theorem:
   "If $a_1(t), a_2(t), \dots, a_{n-1}(t), g(t)$ are <u>continuous</u> on the interval (a,b) containing x_0, then the initial value problem

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = g(t)$$

$$y(x_0) = y_0, y'(x_0) = y_1, \dots, y^{(n-1)}(x_0) = y_{n-1}$$

   has a unique solution for any $y_0, \dots, y_{n-1}$"

   (b) (2 pts) Determine the form of a particular solution to $y'' - y = 3t^2 \sin t + e^t + 7$. You need not solve for the coefficients.
   The homogoeneous part has char. equation $r^2 - 1 = 0$, which has roots ± 1
   $y_p = (At^2 + Bt + C)\cos t + (Dt^2 + Et + F)\sin t + Gte^t + H$

2. (3 pts) Solve the initial value problem $y'' - 6y' + 9y = 0; y(0) = 1, y'(0) = 0$

   Char equation: $r^2 - 6r + 9 = 0$, which factors as $(r-3)^2 = 0$, so $r = 3$ is a repeated root.

   General solution: $y = C_1 e^{3t} + C_2 te^{3t}$

   So $y' = 3C_1 e^{3t} + C_2 e^{3t} + 3C_2 te^{3t}$

   Plugging in 0 to both equations, we get the system $1 = C_1 + 0; 0 = 3C_1 + C_2$, which has solution $C_1 = 1, C_2 = -3$

   Thus, $y = e^{3t} - 3te^{3t}$

3. (4 pts) Find the general solution to $y'' + 4y = \cos(2x)$

   Homogeneous: $r^2 + 4 = 0, r = \pm 2i$, so $y_h = C_1 \cos(2x) + C_2 \sin(2x)$

   We guess $y_p = x(A \sin 2x + B \cos 2x)$

   Then $y_p' = A \sin 2x + B \cos 2x + x(2A \cos 2x - 2B \sin 2x)$

   So $y_p'' = 2A \cos 2x - 2B \sin 2x + 2A \cos 2x - 2B \sin 2x + x(-4A \sin 2x - 4B \cos 2x)$

   Then $y_p'' + 4y_p = 4A \cos 2x - 4B \sin 2x$. We want this to be $\cos 2x$, so we take $A = \frac{1}{4}, B = 0$ and get

   $y = y_h + y_p = C_1 \sin 2x + C_2 \cos 2x + \frac{x}{4} \sin 2x$