Name: ______________________

Section: 9:00-10:00 11:00-12:00

Math 54 Quiz 1 SOLUTIONS

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You have twenty minutes to complete this quiz. You must show your work in order to receive partial credit.

1. (3 pts) Find the solution to the system of equations $\begin{cases} x_1 & + & & - & 3x_3 & = & 8 \\ 2x_1 & + & 2x_2 & + & 9x_3 & = & 7 \\ & & x_2 & + & 5x_3 & = & -2 \end{cases}$

$$\text{Augmented Matrix: } \begin{pmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{pmatrix}$$

$$r_2 = r_2 - 2r_1: \begin{pmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{pmatrix}$$

$$r_2 = r_2 - 2r_3: \begin{pmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -15 \\ 0 & 1 & 5 & -2 \end{pmatrix}$$

$r_2:$ $-5x_3 = 5$, so $x_3 = -1$

$r_3:$ $x_2 + 5(-1) = -2$, so $x_2 = -2 + 5 = 3$

$r_1:$ $x_1 - 3(-1) = 8$, so $x_1 = 5$

The solution is thus $(5, 3, -1)$

2. (3 pts) Find the reduced echelon form of $\begin{pmatrix} 3 & 6 & 0 & 9 \\ 6 & 12 & 2 & 26 \\ -1 & -2 & -1 & -4 \end{pmatrix}$

$$\tfrac{1}{3}r_1, \tfrac{1}{2}r_2 : \begin{pmatrix} 1 & 2 & 0 & 3 \\ 3 & 6 & 1 & 13 \\ -1 & -2 & -1 & -4 \end{pmatrix}$$

$$r_2 = r_2 - 3r_1, r_3 = r_3 - r_1 : \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$r_3 = r_3 - r_2 : \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

$$-\tfrac{1}{3}r_3 : \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$r_2 = r_2 - 4r_3, r_1 = r_1 - 3r_3 \quad \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. (4 pts) The matrices below represent systems of linear equations. For each matrix, determine how many solutions (if any) the corresponding system of equations has.

(a) $\begin{pmatrix} 1 & 3 & 8 & -6 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & -7 & 0 \end{pmatrix}$: one

(b) $\begin{pmatrix} 4 & -1 & 8 & 9 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$: one

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$: none

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$: infinitely many