

Mathematics 54. Spring Semester 2008

Professor: Daniel Tataru

Midterm 1

February 28, 2008, 11:10-12:30

Your Name: _____

TA's Name: _____

Section time: _____

Directions: This is a *closed* book exam. No calculators, cell phones, pagers, mp3 players and other electronic devices are allowed.

Remember: Answers without explanations will not count. You should **show your work**. Solve each problem on its own page. If you need extra space you can use backs of the pages and the extra page attached to your exam paper, but make a note you did so.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	
Grade	

(20) 1. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 1 & 1 & 2 \\ 4 & 1 & 0 & 1 \end{bmatrix}$$

Find $\text{rank } A$, a basis for $\text{Col } A$ and a basis for $\text{Row } A$.

(20) 2. **Problem 2.** Compute (or if undefined say so, explaining why)

a) A^{-1} , $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}$

b) A^{2008} , $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} [1 \ 2 \ 4]$

d) $\det \begin{bmatrix} 7 & 0 & 0 & 4 & 0 \\ 1 & 1 & 2 & 5 & 0 \\ 1 & 4 & 7 & 5 & 2 \\ 3 & 0 & 0 & 2 & 0 \\ 2 & 0 & 1 & 1 & 0 \end{bmatrix}$

(20) 3. a) State Cramer's rule.

b) Find the area of the triangle with vertices $(3, 2)$, $(7, 4)$ and $(4, 5)$.

- (20) 4. Mark each statement True or False. Justify your answers.
- a) $AB = BA$ for all square matrices A, B .

b) The set V of all 3×4 matrices is a vector space of dimension 12.

- (20) 5. Consider the set W of all polynomials p in P_3 with $p(1) = 0$.
- Show that W is a subspace of P_3 .

b) Find a basis in W .

(20) 6. Mark each statement True or False. Justify your answers.

a) If A and B are $n \times n$ matrices so that AB is invertible then BA is invertible.

b) Any 3 linearly independent vectors in R^3 form a basis in R^3 .