Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

1. For each of the following sequences, give a closed form expressing the nth term:

   (a) $(\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots)$

   (b) $(-2, 1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots)$

   (c) $(1, 3, 5, 7, 9, 11, 13, \ldots)$

2. Determine whether $\lim_{n\to\infty} a_n$ converges or diverges. If it converges, find the limit.

   (a) $a_n = \frac{1}{n}$

   (b) $a_n = \frac{n^3+2n+1}{4n^3+n^-n}$

   (c) $a_n = \frac{n!}{2^n}$

   (d) $a_n = n\tan(1/n)$

   (e) $a_n = \frac{\sin n}{n}$

   (f) $a_n = \frac{2^n}{3^{n+1}}$

   (g) $a_n = \ln(n+1) - \ln(n)$

3. Suppose $\{a_n\}$ is a sequence such that $a_n = \frac{P(n)}{Q(n)}$, where P and Q are polynomials of degree r and s, respectively (that is, $P(n) = p_r n^r + p_{r-1}n^{r-1} + \cdots p_0$ and $Q(n) = q_s n^s + q_{s-1}n^{s-1} + \cdots q_0$). For each of the following cases, determine whether $\lim_{n\to\infty}$ exists. When it does, find its value.

   - $r > s$
   - $r < s$
   - $r = s$

4. Prove that if $\lim |a_n| = 0$, then $\lim a_n = 0$. Hint: use the Squeeze Theorem.

5. True/False: For ones that are true, give a short justification. For those that are false, give a counterexample.

   (a) If a_n and b_n have limits, then $a_n + b_n$ has a limit.

   (b) If $a_n + b_n$ has a limit, then a_n or b_n does.

   (c) If a_n and b_n diverge, then $a_n b_n$ diverges.