

Math 1B Discussion Section Problems

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1. Integrals

- (a) Find $\int \frac{\sin x}{\cos^{101} x}$
- (b) Find $\int \frac{1}{x\sqrt{x-1}}$
- (c) Find the partial fraction decomposition for $\frac{x^3 + 2x}{x^3 + 1}$
- (d) Find the length of the curve $y = \frac{1}{x^2}$ between $x = 1$ and $x = 2$
- (e) Determine whether $\int_0^2 \frac{dx}{4x-5}$ is improper. Evaluate it, or show that it diverges.

2. Sequences and Series

- (a) Determine whether each of the following converge or diverge:

- i. $\sum_{n=0}^{\infty} \frac{1000^n}{n!}$
- ii. $\sum_{n=10}^{\infty} \frac{1}{n(\ln(n))^{3/2}}$
- iii. $\sum_{n=1}^{\infty} \left(\sin\left(\pi \frac{n+1}{n}\right) - \sin\left(\pi \frac{n+2}{n+1}\right) \right)$ If it converges, find its value.

- (b) Find a power series representation for each of the following:

- i. $\ln(1+x)$
- ii. $\ln(1-x)$
- iii. $\ln\left(\frac{1+x}{1-x}\right)$
- iv. $\frac{1}{\sqrt{x}}$ centered at $x = 1$

- (c) **Quickly** determine whether each of the following are conditionally convergent, absolutely convergent, or divergent:

- i. $\sum \frac{n^2 - n + 2}{\sqrt[4]{n^{10} + n^5 + 3}}$
- ii. $\sum (-1)^n \frac{1 + e^{-n}}{n}$

- (d) Find the radius and interval of convergence for $\sum_{n=2}^{\infty} (-1)^n \frac{(4x+1)^n}{n}$

3. Differential Equations

- (a) Find the general solution to $y' = \cos^2 y \ln x$
- (b) Solve $y' + (\cos x)y = \sin x \cos x$
- (c) Find the general solution to $x^2 y' = y^2 + 3yx + x^2$
- (d) What is the integrating factor for $y' \cos x = y \sin x + e^x \cos x$?