Math 1B Discussion Section Problems

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1. Recall that the equation for a damped spring is $my'' + cy' + ky = F(t)$, where $F(t)$ is the external force being applied to the system. Now suppose we use this equation to model the motion of the shock absorbers in a bicycle after hitting a pothole. For simplicity, let's assume it's a really old bike that has very little damping force left (ie, c=0). Suppose you try to "ride out" the pothole by gently hopping up and down on your bike right as you hit the bump so the external force you are applying is $F(t) = F_0 \cos \omega_0 t$. Also, let ω be the natural frequency of the shock absorbers.

   (a) If $\omega \neq \omega_0$, use the method of undetermined coefficients to find an equation for the motion of the shock absorbers.

   (b) Now suppose that by some great cosmic coincidence you happen to jump up and down at the same rate as your shock absorbers naturally would have (ie, $\omega_0 = \omega$. Now find a new equation for the motion of your bike.

   (c) What is fundamentally different about the solutions to part (b) versus part (a)?

2. Re-write (ie, re-index) each of the following so that only x^n appears inside the sum:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{2^{n+1}} x^{n+3}$

   (b) $\displaystyle\sum_{n=1}^{100} n x^{n-1}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)}{2} x^{n-2}$

3. Suppose $y = \sum_{n=0}^{\infty} c_n x^n$. Write y' and y'' as power series involving only x^n

4. Let $y = \sum_{n=0}^{\infty} c_n x^n$. In each of the following expersiions, substitute this power series in for y and write the result as one sum involving only x^n

   (a) $y' - 6y$

   (b) $xy'' - y$

   (c) $y'' + xy' + y$

5. (a) Use power series to find the general solution to $y'' + xy' + y = 0$

   (b) Solve the intial value problem $y'' + xy' + y = 0; \;\; y(0) = 0; y'(0) = 1$

6. One of the hardest parts about using power series to solve these differential equations is in finding the pattern in the coefficients. For each of the following, "unwind" the recurrence relation:

   (a) $(n+1)a_{n+1} = a_n$ Write a_n in terms of a_0

   (b) $c_{n+2} = -\frac{c_n}{(n+1)(n+2)}$ Write c_n in terms of c_0 or c_1

   (c) $n^2 a_n + a_{n-2} = 0$ and $a_1 = 0$ Write a_n in terms of a_0

7. Use power series to find the general solution to $y' - y = 0$. Is the answer what you expected?