Math 1B Discussion Section Problems

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1. Use variation of parameters to find the general solution to:
   (a) $y'' - 2y' + y = \frac{e^x}{1+x^2}$
   (b) $y'' - y = \frac{1}{x}$
   (c) $y'' + 4y' + 4y = \frac{e^{-2x}}{x^3}$

2. (a) Does the method of variation of parameters always work? If yes, why? If not, what could go wrong?
   (b) Find an example of a differential equation that cannot be solved with variation of parameters. (Hint: try to work backwards so that $y_1 = 1$ and $y_2 = e^x$)

3. It turns out that integrating factors are really just a special case of variation of parameters. Here we'll explore why by finding the general solution to $y' + P(x)y = Q(x)$. For this entire problem, try to forget that you ever learned about integrating factors.
   (a) Find the general solution to $y' + P(x)y = 0$. Call this solution $R(x)$ and note that it may include $\int P(x)$ somewhere.
   (b) You should have $R(x) = Ce^{-\int P(x)}$. Since we're doing variation of parameters, replace the constant C by an arbitrary function $u(x)$ and show that $u(x)e^{-\int P(x)}$ will be a solution to $y' + P(x)y = Q(x)$ if $u' = Q(x)e^{\int P(x)}$.
   (c) Show that the general solution to $y' + P(x)y = Q(x)$ is given by $y = e^{-\int P(x)} \int Q(x)e^{\int P(x)}$

4. (a) Show that if a, b, c are all greater than 0, then all solutions to $ay'' + by' + cy = 0$ have the property that $\lim_{x\to\infty} y(x) = 0$
   (b) If $a > 0, c > 0$ but $b = 0$, show that the result from part (a) is no longer true, but that all solutions are bounded as $x \to \infty$
   (c) If $a > 0, b > 0$ and $c = 0$, show that all solutions approach some constant (not necessarily 0) as $x \to \infty$. Determine this constant in terms of $y(0)$, which we'll call y_0, and $y'(0)$, which we'll call y'_0