Math 1B Discussion Section Problems

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1. Find the general solution to $y'' - 6y' = -9y$
2. Solve the initial value problem $y'' - 6y' + 13y = 0$ where $y(0) = 4$ and $y'(0) = 0$
3. Consider the equation $x^2 + 6x + 13 = 0$
   (a) Solve it. Call the two solutions r and s.
   (b) Compute s^2. Write your answer in the form $a + bi$
   (c) Find s^{-2}. Again, write your answer in the form $a + bi$
   (d) Plot $s^2, 6s$,and 13 as vectors in the Complex plane. Show geometrically that these vectors add to 0.
4. Suppose $z = x + iy$ is a complex number.
   (a) Draw $x + iy$ as some point in the complex plane. (It's probably easiest if you put it somewhere in the first quadrant)
   (b) Find the distance between z and the origin.
   (c) Find the angle between z and the x-axis. Hint: draw a right triangle
   (d) How can you use parts (b) and (c) to write z as $re^{i\theta}$?
5. (a) Find an expression for $\dfrac{a+bi}{c+di}$. Write your answer in the form $x + iy$.
   (b) Now find an expression for $\dfrac{re^{i\theta}}{se^{i\varphi}}$. Write your answer in the form xe^{iy}
6. Like most things, it turns out that the temperature function $T(t)$ of a Turkey in an oven obeys the differential equation $\frac{dT}{dt} = k(T_{oven} - T)$, where k is some arbitrary constant and T_{oven} is the (constant) temperature of the oven.
   (a) Solve the differential equation to get an explicit formula for $T(t)$ in terms of k, T_{oven} and some arbitrary constant C
   (b) Suppose you put a $20°C$ Turkey in a $205°$ oven and after 30 minutes it has warmed to $32°$. Use the given initial condition and the temperature after 30 mintutes to find actual values for k and C
   (c) How long does it take before the turkey reaches a well-done $82°$?
7. (a) Show that $y = \cos(x + A)$ is a solution to $y'' = -y$
   (b) Explain why part (a) means we must have that $\cos(x + A) = C_1 \cos(x) + C_2 \sin(x)$ for some constants C_1 and C_2
   (c) Find the values of C_1 and C_2. Hint: You need two equations since you have two unknowns. For one equation, plug in 0 to both sides. For another, take the derivative of both sides and then plug in 0.
8. Show that if $z = r(\cos\theta + i\sin\theta)$, then $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$. Hint: use Euler's formula to write z in a more convenient form