Math 1B Discussion Section Problems

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1. What is the definition of two functions being linearly independent?
2. For each of the following pairs of functions, determine whether they are linearly independent or dependent:
   (a) $f(x) = e^x,\ g(x) = e^{-x}$
   (b) $f(x) = e^x,\ g(x) = e^{x+1}$
   (c) $f(x) = xe^{2x},\ g(x) = e^{2x}$
   (d) $f(x) = \ln(x^3),\ g(x) = \ln(x^\pi)$
   (e) $f(x) = \sin(x),\ g(x) = \cos(x)$
3. Which of the following second order differential equations are linear? Homogeneous?
   (a) $e^x y'' + \cos(3x^2)y' + 3y = 0$
   (b) $y'' + 3y' + 7y = \cos x$
   (c) $y'' + 3y' + y^2 = 0$
   (d) $\tan(y'') + \cos(x)y' = e^x$
4. Find the characteristic equation for each of the following ODEs:
   (a) $y'' + y' - 6y = 0$
   (b) $y'' = -y$
   (c) $4y'' + 3y' - y = 0$
5. Find the general solution to $y'' + 3y' - 18 = 0$
6. Solve the initial value problem $y'' + 3y' - 10y = 0$ with $y(0) = 1$ and $y'(0) = 3$
7. Solve the boundary value problem $y'' = y$ with $y(0) = 0$ and $y(2) = 2$
8. Most of the time (especially in this class), determining whether two functions are linearly independent can be done by just staring at them and making an educated guess. However, sometimes things are more complicated and sometimes you need actual proof that they are linearly independent. One method for doing this is to compute the Wronskian, $W(f, g) = fg' - f'g$, as it turns out that two functions are linearly dependent if and only if $W(f, g) = 0$.
   (a) Use the Wronskian to show that e^{at} and e^{bt} are linearly independent whenever $a \neq b$
   (Hint: a function $y(x)$ is equal to 0 if and only if $y(x) = 0$ for every value of x)
   (b) Show that e^{at} and te^{at} are linearly independent for any real number a
   (c) Show that $\cos(at)$ and $\sin(at)$ are linearly independent for any real number $a \neq 0$.