Math 1B Discussion Section Problems

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November 13, 2007

1. Suppose you just bought a room-temperature can of Coke and want to cool it as quickly as possible. In particular, you have two options: you could put it in the freezer, or you could put it in a bucket of icewater. On the one hand, the freezer is colder than the ice water, but on the other, ice water conducts heat much better than freezer air. This problem will investigate which method is more efficient.

   Some background physics: the amount of heat in the can is given by the equation $H(t) = mcT(t)$, where $T(t)$ is the tempurature at time t, m is the mass of the can (including fluid), and c is a constant referred to as the heat capacity. Also, the rate at which heat enters the air/ice water is $hA(T - t_\infty$, where A is the surface area of the can, h is a constant known as the convection coefficient, T is temperature of the can, and T_∞ is the temperature of the air/ice water.

   (a) Use the information above and the fact that any heat leaving the can must enter the surrounding air/icewater to show that $mcT' = -hA(T - T_\infty)$ is a differential equation for the temperature of the can. You may assume T_∞ is kept constant.

   (b) Solve your differential equation from part (a). Hint: while this equation is linear, there is a much simpler way to solve it.

   (c) It turns out that $c = 4000$ is a pretty good choice for a typical can of soda, that $h = 40$ for freezer air or 160 for ice water, and that $T_\infty = -20$ for the freezer and 0 for the ice water. Suppose the can was originally at 30^o and that $m = .13$, $A = .033$ for a typical can. Using this information, find equations for the temperature of the can as a function of time. Note: the units for all these constants are in terms of meters, seconds, kgs, and joules.

   (d) How cold will the soda be after one hour (3600 seconds) in the freezer? In the ice water?

   (e) If you put a can in the freezer and in the ice water at the same time, is there ever a time in the future where they will be the same temperature again? Hint: think about the Intermediate Value Theorem.

   (f) Why is T_∞ an appropriate choice of variable name for the icewater/air temperature?

2. Psychologists often study how long it takes people to learn to do a certain skill (such as writing differential equations!) and have found that the rate at which people learn is proportional to the difference between their current performance level, which we'll call P, and their maximum performance level, which is a constant we'll call M.

   (a) Write a differential equation that models this idea.

   (b) Sketch some solution curves for your differential equation

   (c) Solve your differential equation using the initial condition $P(0) = P_0$

3. (More about circuits!) Suppose we have a setup like what's drawn on the blackboard. Then the voltage drop accross a resistor is IR and Q/C accross a capicitor, and physics tells us that $I = Q'$. Thus, we can write the differential equation $RQ' + \frac{1}{C}Q = E(t)$, where R is the resistance, C is the capicitance, Q is the charge, and $E(t)$ is the voltage being pushed through the system.

   (a) Suppose the resistance is 5Ω, the capacitance is $.05F$ and a battery is supplying a constant $60V$. If the initial charge is $Q(0) = 0$, find an equation for the charge at time t.

   (b) Using the fact that $I = Q'$, find an equation for the current.

   (c) Re-solve part (a) supposing the battery is replaced by an AC wall outlet that supplies voltage at $E(t) = 120\sin(\frac{2\pi}{60}t)$ Volts.