Math 1B Discussion Section Problems

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1. What is the difference between a Taylor Series and a Maclaurin Series?
2. Find a Taylor Series expansion for each of the following functions and find the associated radius of convergence.
   (a) $f(x) = \sin x$, centered at $x = \pi/2$
   (b) $\sin^2(x)$, centered at 0. Hint: $2\sin x \cos x = \sin(2x)$
   (c) $\frac{1}{\sqrt{x}}$, centered at 4.
   (d) $\frac{1}{\sqrt[4]{16+x}}$ (Hint: don't try to take derivatives)
3. We already know that $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$. Using this, find a Maclaurin series for:
   (a) $\sin(x^3)$
   (b) $x\sin(x^3)$
   (c) Can you use similar techniques to find a Taylor Series for $\sin(\sqrt{x+1})$? How about for $\cos(\sqrt{x+1})$?
4. (All about 0!)
   (a) How can you find 5! if you already know 4!?
   (b) How about 2! if you know 1!?
   (c) Why do the above two examples give a good hint as to why we should say $0! = 1$
5. (More proof that infinity is very, very strange)
   (a) What is $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$
   (b) Find a power series for $\frac{x}{(1-x)^2}$
   (c) Use your answer to (b) to find $\sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n$
6. Let $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$.
   (a) Sketch a graph of this function.
   (b) Find $f'(0)$ by using the definition of derivative.
   (c) It can be shown (though it's rather difficult) that $f^{(n)}(0)$ exists and is 0 for all n. What does this mean for the Taylor series of f at 0? What is its radius of convergence?
   (d) For what values of x does $f(x)$ equal this Taylor Series?