

# Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems and not just the answers.

1. Determine whether  $\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$  converges. Does it converge absolutely?
2. Find the radius and interval of convergence for each of the following power series:

(a)  $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(b)  $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

(c)  $\sum_{n=1}^{\infty} x^n \ln\left(1 + \frac{1}{n}\right)$

(d)  $\sum_{n=1}^{\infty} \frac{(3x-1)^n}{n4^n}$

3. Given the interval of convergence for  $\sum a_n x^n$ , can you determine the radius? How about the other way around?
4. True/False: For those that are true, explain why. For those that are false, give a counterexample.
  - (a) If  $\sum a_n (-3)^n$  converges, then  $\sum a_n 2^n$  converges.
  - (b) If  $\sum a_n (-3)^n$  converges, then  $\sum a_n 3^n$  converges.
  - (c) If  $\sum a_n x^n$  has a positive radius of convergence, then  $\lim a_n = 0$
5. For each  $n$ , let  $f_n(x) = \frac{\sin(nx)}{n^2}$ . Show that  $\sum f_n(x)$  converges for all  $x$ . Is the same true of the series of derivatives,  $\sum f'_n(x)$ ?