Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems and not just the answers.

1. Explain the difference between a sequence and a series. In each case, what techniques do you know for determining whether or not it converges?
2. Determine whether each of the following series converge or diverge.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+2n^2+3}$
   (b) $\displaystyle\sum_{n=3}^{\infty} \frac{n}{n^3-2n^2-3}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2+\ln n}$
   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$
   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{3n^3+4n}{2^n(n^3+6n^2)}$
3. For each of the following, determine (a) whether it is an alternating series and (b) if it converges.
   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\cos(\pi n)}{\sqrt{n}}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3^n}$
   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$
4. True/False. For those that are True, prove it. For those that are false, give a counterexample.
   (a) If $\sum a_n$ converges and each $a_n \neq 0$, then $\sum \frac{1}{a_n}$ diverges.
   (b) If $\sum a_n$ converges and each $a_n \geq 0$, then $\sum a_n^2$ converges.
   (c) If $\sum a_n$ converges, then $\sum \sqrt{a_n}$ diverges.
   (d) If $\sum a_n$ converges, then $\sum \sqrt{a_n}$ converges.