Math 1B Discussion Section SELECTED Solutions

Rob Bayer

October 9, 2007

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems and not just the answers.

1. (e) If we look at important parts, we see that this behaves basically like $\frac{n^3}{2^n n^3}$, so we'll try comparing with $\frac{1}{2^n}$. I don't see any nice inequality to get, so we'll do limit comparison:

$$\lim_{n\to\infty} \frac{\frac{3n^3+4n}{2^n(n^3+6n^2)}}{1/2^n} = \lim \frac{3n^3+4n}{n^3+6n^2} = 3$$

So the two series do the same thing. But $\sum \frac{1}{2^n} = \sum \left(\frac{1}{2}\right)^n$, which is a geometric series with $r = 1/2$ and thus converges. Therefore $\sum_{n=2}^{\infty} \frac{3n^3+4n}{2^n(n^3+6n^2)}$ converges too.

2. (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

   This is alternating with $b_n = 1/n$, which is both decreasing and goes to zero, so it is convergent.

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\cos(\pi n)}{\sqrt{n}}$

   This is actually NOT alternating, since $\cos(\pi n) = (-1)^n$, so this is just $\sum \frac{1}{\sqrt{n}}$, which is a divergent p-series.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3^n}$

   This is alternating with $b_n = \frac{n}{3^n}$. Let $f(x) = \frac{x}{3^x}$. As $x \to \infty$, this becomes $\frac{\infty}{\infty}$, so we use L'Hospital's to get $\lim f(x) = \lim \frac{1}{3^x \ln 3} = 0$. Also, $f'(x) = \frac{3^x - x3^x \ln(3)}{3^{2x}}$. Note that $x3^x \ln(3) > 3^x$, so $f' < 0$ meaning f is decreasing and thus b_n is too. By the alternating series test, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3^n}$ converges.

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$

   This is not technically alternating until after $n = 2$, but I would still probably call this an alternating series. It is NOT convergent however since $\lim \cos(\pi/n) = 1$, and thus $\lim(-1)^n \cos(\pi/n)$ doesn't exist. In particular, it is not 0, so the series diverges by the Divergence Test.

3. True/False. For those that are True, prove it. For those that are false, give a counterexample.

   (a) If $\sum a_n$ converges and each $a_n \neq 0$, then $\sum \frac{1}{a_n}$ diverges.

   True. Since $\sum a_n$ converges, $\lim a_n = 0$ and thus $\lim \frac{1}{a_n} \neq 0$.

   (b) If $\sum a_n$ converges and each $a_n \geq 0$, then $\sum a_n^2$ converges.

   True. As above $\lim a_n = 0$, so eventually $a_n < 1$ and then $a_n^2 < a_n$, so $\sum a_n^2$ converges by the comparison test.

(c) If $\sum a_n$ converges, then $\sum \sqrt{a_n}$ diverges.
False. $a_n = \frac{1}{n^4}$ is one possible counterexample.

(d) If $\sum a_n$ converges, then $\sum \sqrt{a_n}$ converges.
False. $a_n = \frac{1}{n^2}$ is one possible counterexample.