Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems and not just the answers.

1. For each of the following, determine whether the sequence is (a)monotone, (b)bounded, (c)convergent.
   (a) $a_n = \frac{1}{n(n+1)}$
   (b) $a_n = ne^{-n}$
   (c) $a_n = \sin n$
   (d) $a_n = \sin(2\pi n)$
   (e) $a_n = \sin\left(\frac{1}{n}\right)$
2. Determine whether each of the following series are convergent or divergent. For those that are convergent, find the sum when possible:
   (a) $\sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{2n}}$
   (b) $\sum_{n=1}^{\infty} \arctan(n+1) - \arctan(n)$
   (c) $\sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$
   (d) $\sum_{n=1}^{\infty} \sqrt[n]{2}$
3. (Applications)
   (a) (Zeno's Paradox) Suppose you are 1 meter away from a wall and want to walk up and touch it. Then you must first go half the distance to the wall, which takes some positive amount of time, then half of the remaining distance, then half of what still remains, etc, etc. Show that despite this, it only takes a finite amount of time to walk across the room. Assume you can move at 1m/s.
   (b) Prove that $.\overline{9} = 1$
4. (A Preview Of Power Series) For which values of x do each of the following converge?
   (a) $\sum_{n=0}^{\infty} x^n$
   (b) $\sum_{n=0}^{\infty} x^n 2^n$
   (c) $\sum_{n=1}^{\infty} \frac{x}{n}$
   (d) $\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^{2n+1}}$
5. True/false. For those that are true, provide a brief explanation/intuition of why. For those that are false, find a counterexample:
   (a) If $\sum a_n$ converges, then $\sum \sqrt{a_n}$ converges
   (b) If a_n is positive for all n, and each partial sum is less than 10^4, then $\sum_{n=0}^{\infty} a_n$ converges
   (c) If $a_n < b_n$ for all n and both sequences converge, then $\lim a_n < \lim b_n$
   (d) If s_n is the sequence of partial sums for the sequence a_n and $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} s_n$ exists.