Name: ______________________________
Section: 8:00-9:30 11:00-12:30

Math 1B Quiz 8 SOLUTIONS

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You have twenty minutes to complete this quiz. You must justify all your answers.

1. (4 pts) Starting from the geometric series, find a power series representation for $\frac{1}{(1-x^2)^2}$.
   Hint: Start with a power series for $\frac{1}{(1-t)^2}$

$$\frac{1}{(1-t)^2} = \frac{d}{dx}\frac{1}{1-t} = \frac{d}{dx}\sum_{n=0}^{\infty} t^n = \sum_{n=1}^{\infty} nt^{n-1}$$

   So we can make the substitution the substitution $t = x^2$ to get:

$$\frac{1}{(1-x^2)^2} = \sum_{n=1}^{\infty} nx^{2n-2}$$

2. (2 pts) Suppose $\sum_{n=0}^{\infty} a_n(-2)^n$ converges and $\sum_{n=0}^{\infty} a_n 2^n$ diverges. Find the interval and radius of convergence for $\sum_{n=0}^{\infty} a_n(x-2)^n$

   The given series are equivalent to saying that the power series converges at $x = 0$ and diverges at $x = 4$. Because it diverges at $x = 0$, we know $R \leq 2$. Because it converges at $x = 4$, we know the $R \geq 2$, so we must have $R = 2$. Then since we are given the endpoints, we know the interval is $[0, 4)$

3. (4 pts) Find a Taylor series for $f(x) = \ln x$, centered at $x = 2$. Express your answer in $\sum$-notation

n	$f^{(n)}(x)$	$f^{(n)}(2)$
0	$\ln x$	$\ln 2$
1	$\frac{1}{x}$	$\frac{1}{2}$
2	$-\frac{1}{x^2}$	$-\frac{1}{2^2}$
3	$\frac{2}{x^3}$	$\frac{2}{2^3}$
4	$-\frac{3\cdot 2}{x^4}$	$-\frac{3\cdot 2}{2^4}$

   So then our Taylor series is $\ln 2 + \frac{1/2}{1!}(x-2) + \frac{1/4}{2!}(x-2)^2 - \frac{2/8}{3!}(x-2)^3 + \cdots$

   Then in sigma notation, we have $\ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1}\frac{(n-1)!}{2^n n!}(x-2)^n = \ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1}\frac{(x-2)^n}{n2^n}$