Name: ______________________________
Section: 8:00-9:30 11:00-12:30

Math 1B Quiz 7
October 18, 2007
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You have twenty minutes to complete this quiz. You must justify all your answers.

1. (6 pts) For each of the following problems, determine whether the series diverges, converges conditionally, or converges absolutely.

   (a) $\sum_{n=1}^{\infty} \frac{ne^n}{(2n)!}$
   We'll use the ratio test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)e^{n+1}}{(2n+2)!}\frac{(2n)!}{ne^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)e}{n(2n+2)(2n+1)}\right| = 0 < 1$$

   So by the ratio test, the series converges absolutely.

   (b) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$
   We'll use the absolute convergence test. Ie, we'll figure out whether or not $\sum \frac{|\cos n|}{n^2}$ converges. But we know this converges by the comparison test since $\frac{|\cos n|}{n^2} \leq \frac{1}{n^2}$. Thus, $\sum \left|\frac{\cos n}{n^2}\right|$ converges, so $\sum \frac{\cos n}{n^2}$ converges absolutely.

2. (4 pts) Determine the interval and radius of convergence for the power series: $\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n\sqrt{n}}$

   As always, we'll do the ratio test:

$$\lim_{n\to\infty}\left|\frac{(x-1)^{n+1}}{2^{n+1}\sqrt{n+1}}\frac{2^n\sqrt{n}}{(x-1)^n}\right| = \lim_{n\to\infty}\left|\frac{(x-1)\sqrt{n}}{2\sqrt{n+1}}\right| = \frac{|x-1|}{2}$$

   So we need $|x-1|/2 < 1$, which is the same as $|x-1| < 2$. Thus, the radius of convergence is 2. To find the interval, we re-write this inequality as $-2 < x-1 < 2$, so $-1 < x < 3$ and now we only need to test the endpoints:

   ($x = -1$): The series is now just $\sum \frac{(-2)^n}{2^n\sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}}$, which converges by the alternating series test.

   ($x = 3$): We have $\sum \frac{2^n}{2^n\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$, which is a divergent p-series.

   Therefore, the interval of convergence is $[-1, 3)$ and as we said before, $R = 2$.