Name: ______________________________
Section: 8:00-9:30 11:00-12:30

Math 1B Quiz 6 SOLUTIONS

October 11, 2007
GSI: Rob Bayer

You have twenty minutes to complete this quiz. You must justify all your answers.

For each problem, determine whether the series converges or diverges:

1. (3 pts) $\sum_{n=2}^{\infty} ne^{-n^2}$. DO NOT use any of the comparison tests or the ratio test.

   Let $f(x) = xe^{-x^2}$, so f is positive and decreasing so we can apply the integral test. We will make the substitution $u = -x^2$, so $du = -2xdx$ and we get:

$$\int_2^{\infty} xe^{-x^2}dx = \lim_{t\to\infty} \int_2^t xe^{-x^2}dx = -\frac{1}{2}\lim \int_{-4}^{-t^2} e^u du = -\frac{1}{2}\lim(e^{-t^2} - e^{-4}) = \frac{1}{2e^4}$$

   So by the integral test, $\sum ne^{-n^2}$ converges.

2. (3 pts) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+1}$

   This looks mostly like $\frac{n^2}{n^3} = 1/n$, so we'll use the limit comparison test with $b_n = 1/n$:

$$\lim_{n\to\infty} \frac{\frac{n^2-1}{n^3+1}}{\frac{1}{n}} = \lim \frac{n^3-1}{n^3+1} = 1 > 0$$

   so $\sum a_n$ and $\sum b_n$ do the same thing. Then since $\sum \frac{1}{n}$ is a divergent p-series, $\sum \frac{n^2-1}{n^3+1}$ diverges by the limit comparison test.

3. (4 pts) $\sum_{n=1}^{\infty} \frac{1+\cos^2\sqrt{n}}{n^3}$

   $\cos^2(\sqrt{n}) \leq 1$, so $\frac{1+\cos^2\sqrt{n}}{n^3} \leq \frac{2}{n^3}$. Furthermore, $\sum \frac{2}{n^3}$ is a convergent p-series ($p = 3 > 1$), so by the comparison test, $\sum_{n=1}^{\infty} \frac{1+\cos^2\sqrt{n}}{n^3}$ converges too.