Name: ____________________________

Section: 8:00-9:30 11:00-12:30

Math 1B Quiz 5 SOLUTIONS

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You have twenty minutes to complete this quiz. You must show your work.

1. (3 pts) Determine whether the sequence $a_n = \dfrac{2+\sin(n)}{n^2+\ln n}$ converges or diverges. If it converges, find its value.

   Since $-1 \leq \sin n \leq 1$, we have $\frac{1}{n^2+\ln n} \leq \frac{2+\sin n}{n^2+\ln n} \leq \frac{3}{n^2+\ln n}$. Both the left and right hand side of this inequality go to 0 as $n \to \infty$, so by the squeeze theorem, the original sequence must also.

2. (4 pts) Determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ converges or diverges. If it converges, find its value.

   We start with partial fractions: $\frac{2}{n^2-1} = \frac{A}{n-1} + \frac{B}{n+1}$, so $2 = A(n+1) + B(n-1)$ which gives the equations $A+B=0$, $A-B=2$. This has $A=1, B=-1$, for a solution, so $\frac{2}{n^2-1} = \frac{1}{n-1} - \frac{1}{n+1}$.

   Let's try a few partial sums:

$$\begin{aligned}
s_2 &= \frac{1}{1} - \frac{1}{3} \\
s_3 &= 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} \\
s_4 &= 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} \\
&= 1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{5} \\
&\vdots \\
s_n &= 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}
\end{aligned}$$

   So $\lim_{n\to\infty} s_n = \dfrac{3}{2}$ and the sum is convergent and has value $\frac{3}{2}$

3. (3 pts) True/False. For those that are true, give a (short) explanation of why and for those that are false, give a counterexample.

   (a) If a_n and b_n are sequences such that a_n and $a_n + b_n$ both converge, then b_n converges.

   True If a_n and $a_n + b_n$ are both convergent, then $(a_n + b_n) - a_n$ is convergent, but this is just b_n

(b) Every bounded sequence converges.

False $a_n = (-1)^n$ is one possible counterexample.

(c) If s_n is the sequence of partial sums for $\sum_{n=1}^{\infty} a_n$ and $\lim_{n\to\infty} s_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

True This is actually the definition of $\sum a_n = 0$