

Name: _____
Section: 8:00-9:30 11:00-12:30

Math 1B Quiz 4

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You have twenty minutes to complete this quiz. You must show your work.

1. (3 pts) Determine whether the integral $\int_1^\infty \frac{\cos^2 x}{1+x^2} dx$ converges or diverges.

Since $\cos^2 x \leq 1$ and $1+x^2 > x^2$, $\frac{\cos^2 x}{1+x^2} \leq \frac{1}{x^2}$.

Furthermore, $\int_1^\infty \frac{1}{x^2} dx$ converges since it is a p-integral with $p = 2$.

So by the Comparison Test, $\int_1^\infty \frac{\cos^2 x}{1+x^2} dx$ converges.

2. (3 pts) Find $\int_0^2 \frac{x}{x^2-1} dx$, or show that it does not exist.

We note that this is discontinuous at $x = 1$, so we write $\int_0^2 \frac{x}{x^2-1} dx = \int_0^1 \frac{x}{x^2-1} dx + \int_1^2 \frac{x}{x^2-1} dx$. Note that $\int \frac{x}{x^2-1} dx = 1/2 \ln |x^2-1|$ (if you don't see this, substitute $u = x^2-1$), so

$$\int_0^1 \frac{x}{x^2-1} dx = \lim_{t \rightarrow 1^-} \int_0^t \frac{x}{x^2-1} dx = \lim_{t \rightarrow 1^-} (\ln |t^2-1| - \ln |-1|) = -\infty$$

And thus the original integral diverges.

3. (4 pts) Suppose we want to use the Trapezoid rule to estimate $\int_1^2 \frac{1}{x} dx$ and want an error of less than 10^{-5} . How many subintervals (ie, how big of an n) do we need to guarantee this?

$f(x) = 1/x$, so $f' = -1/x^2$ and $f'' = 2/x^3$. On $(1, 2)$, this is always less than 2, so we pick $K = 2$

Then we want

$$\begin{aligned} |E_T| &\leq \frac{2(2-1)^3}{12n^2} \leq 10^{-5} \\ \frac{10^5}{6} &\leq n^2 \\ \sqrt{\frac{10^5}{6}} &\leq n \end{aligned}$$

So we can pick any integer greater than $\sqrt{\frac{10^5}{6}}$.