Name ________________________________

**Math 1B Quiz 2 SOLUTIONS**

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Find/Evaluate each of the following. You must show your work.

1. (3pts) $\int \frac{\ln(x)}{x^2} dx$

   We do this by parts: $u = \ln(x), dv = 1/x^2 dx$, so $du = 1/x, v = -1/x$ and we have:

$$\begin{aligned} \int \frac{\ln(x)}{x^2} dx &= -\frac{\ln(x)}{x} - \int -\frac{1}{x^2} dx \\ &= -\frac{\ln(x)}{x} + \frac{-1}{x} + C \end{aligned}$$

2. (3pts) $\int_0^{\pi/4} \tan^5 x \sec x dx$ We first note that

$$\int \tan^5 x \sec x dx = \int (\tan^2 x)^2 \sec x \tan x dx = \int (\sec^2 x - 1)^2 \sec x \tan x dx$$

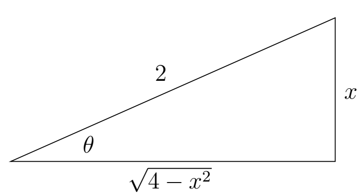
   We make the substitution $u = \sec x$, so $du = \sec x \tan x$ and our bounds of integration become 1 and $\sqrt{2}$

$$\begin{aligned} \int_1^{\sqrt{2}} (u^2-1)^2 du = \int_1^{\sqrt{2}} u^4 - 2u^2 + 1 du &= \frac{u^5}{5} - \frac{2u^3}{3} + u \Big|_1^{\sqrt{2}} \\ &= \frac{\sqrt{2}^5}{5} - \frac{2\sqrt{2}^3}{3} + \sqrt{2} - \left(\frac{1}{5} - \frac{2}{3} + 1\right) \\ &= \frac{7\sqrt{2} - 8}{15} \end{aligned}$$

3. (4pts) $\int \frac{x-1}{\sqrt{4-x^2}} dx$ We make the substitution $x = 2\sin\theta$, so $dx = 2\cos\theta d\theta$:

$$\int \frac{x-1}{\sqrt{4-x^2}} dx = \int \frac{(2\sin\theta - 1)2\cos\theta}{\sqrt{4 - 4\sin^2\theta}} d\theta = \int \frac{(2\sin\theta-1)2\cos\theta}{2\cos\theta} d\theta = \int 2\sin\theta - 1 = -2\cos\theta - \theta + C$$

   To get back to x's, we note that our original substitution could be written as $\frac{x}{2} = \sin\theta$. So we draw a triangle to find $\cos\theta = \frac{\sqrt{4-x^2}}{2}$ and then note that $\theta = \sin^{-1}\frac{x}{2}$:




   So our final answer is just:

$$-\sqrt{4-x^2} - \sin^{-1}\left(\frac{x}{2}\right) + C$$