Name: ____________________
Section: 8:00-9:30 11:00-12:30

# Math 1B Quiz 10 SOLUTIONS

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You have twenty minutes to complete this quiz. You must justify all your answers.

1. (3 pts) Find the solution to the initial value problem $y' = 2 + 2x^2 + y + x^2y;\ y(0) = 4$

   $y' = 2 + 2x^2 + y + x^2y = 2(1 + x^2) + y(1 + x^2) = (2 + y)(1 + x^2)$

$$\begin{aligned}\frac{dy}{2+y} &= (1+x^2)dx \\ \ln(2+y) &= x + \frac{x^3}{3} + C \\ 2+y &= Ce^{x+\frac{x^3}{3}} \\ y &= Ce^{x+\frac{x^3}{3}} - 2\end{aligned}$$

   To solve for $C$, we plug in $x = 0, y = 4$ to get $4 = Ce^0 - 2$, so $C = 6$ and the solution is just

$$y = 6e^{x+\frac{x^3}{3}} - 2$$

2. (3 pts) Find the general solution to $xy' - 2y = x^2$

   Divide through by $x$ to get $y' - \frac{2}{x}y = x$, so $P(x) = -2/x$, $Q(x) = x$ Then we have:

$$\begin{aligned}I &= e^{\int -\frac{2}{x}} = e^{-2\ln(x)} = x^{-2} \\ y &= \frac{1}{I}\int IQ \\ &= x^2 \int \frac{1}{x} \\ &= x^2(\ln x + C) \\ &= x^2 \ln(x) + Cx^2\end{aligned}$$

3. (4 pts) Find the general solution to $x^2y' = y^2 + 3yx + x^2$

   Dividing by $x^2$ gives $y' = \left(\frac{y}{x}\right)^2 + 3\frac{y}{x} + 1$, so we make the substitution $v = \frac{y}{x}$. Then $y = vx$, so $y' = v + xv'$ and we get:

$$
\begin{aligned}
v + xv' &= v^2 + 3v + 1 \\
v' &= \frac{v^2 + 2v + 1}{x} \\
\frac{dv}{(v+1)^2} &= \frac{dx}{x} \\
-\frac{1}{v+1} &= \ln(x) + C \\
v + 1 &= -\frac{1}{\ln(x) + C} \\
v &= \frac{-1}{\ln(x) + C} - 1 \\
y &= \frac{-x}{\ln(x) + C} - x
\end{aligned}
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