Math 1B Test 1 Topics & Review[1]
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**NOTE:** In order to save space and to avoid getting bogged down in details, I've left out of a lot of notation stuff (like dx in integrals, the (x) in f(x), etc). You should always remember these things! Do as I say, not as I do...

1. Substitution rule (u-substitutions)
   - **When to use:** When you can pick $u$ so that its derivative is somewhere else in the function.
   - **How to use:** Pick $u$ and calculate $\mathrm{d}u$. Plug these in and do the integral as normal.
   - **Reminders, Tips, and Tricks:**
     - Don't forget to plug back in $u$ when you're done–if the original problem was in terms of $x$, your answer should be too!
     - This is often a small step in a larger problem.
     - Best bets for picking $u$ are things whose derivative is somewhere else in the function. Square roots can also sometimes be a good choice (see Quiz 3, Problem 1)
2. Integration by Parts
   - **When to use:** Use when you can't see a u-sub, especially if there are two things multiplied together and one of them is a "good" choice for $u$ (see tips and tricks)
   - **How to use:** $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$
   - **Reminders, Tips, and Tricks:**
     - You want to pick $u$ so that it gets simpler when you takes its derivative. A good rule for this is **LIATE**: Logs, Inverse trig, Algebra (meaning polynomials), Trig, and Exponentiation (like $e^x$).
     - This should be one of your last resorts (unless you can clearly see that its a by parts problem)
3. Trigonometric Integration
   - **When to use:** For integrals of the form $\sin^m x \cos^n x$ or $\sec^m x \tan^n x$
   - **How to use:** For $\sin^m x \cos^n x$, if one of them is odd, save one factor of that and convert the rest.
     For $\sec^m x \tan^n x$, if secant is even, save one $\sec^2$ and convert the rest to tan. If tangent is odd, save one $\sec x \tan x$ and convert the rest of the tangents to secants.
   - **Reminders, Tips, and Tricks:** While you can put a lot of the trig identities on your "cheat sheet", you should know $\tan^2 x + 1 = \sec^2 x$ and $\sin^2 + \cos^2 = 1$ by heart!
4. Trigonometric Substitution
   - **When to use:** When the integral contains one of the three types of square roots given in your book and a u-sub doesn't work.
   - **How to use:** Set $x = \sin\theta, \tan\theta$ or $\sec\theta$, find $\mathrm{d}x$ in terms of $\mathrm{d}\theta$, and simplify using trig identities
   - **Reminders, Tips, and Tricks:**
     - $\sqrt{a^2 - x^2} \to x = a\sin(\theta)$, $\sqrt{a^2 + x^2} \to x = a\tan(\theta)$, $\sqrt{x^2 - a^2} \to x = a\sec(\theta)$
     - When you're done, use the right triangle rules (sohcahtoa) to get rid of all the $\theta$s. If you're left with just a plain old $\theta$, use your original substitution formula to express $\theta$ as some inverse trig function of $x$.
     - Don't jump immediately to trig-sub: problems like $\int \frac{2x}{\sqrt{x^2+1}}$ can be done with a simple u-sub!

[1]No guarantees this is complete or even accurate, but I tried!

5. Partial Fractions

- **When to use:** When the integral is the division of two polynomials and the one in the denominator factors.
- **How to use:** Four cases:
  (a) Distinct linear factors: $\frac{A}{ax+b}$
  (b) Repeated linear factors: $\frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + ...$
  (c) Distinct quadratic factors: $\frac{Ax+B}{ax^2+bx+c}$
  (d) Repeated quadratic factors: $\frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{(ax^2+bx+c)^2} + ...$
- **Reminders, Tips, and Tricks:**
  - If the numerator is 1 (and the denominator factors), it's almost always a partial fractions problem.
  - The degree of the numerator must be strictly smaller. If not, do long division.
  - To save some time when solving for A, B, etc, plug in values of x that make things disappear.
  - When integrating the quadratic terms, it never hurts to complete the square and possibly break up the integral.
  - Common mistake 1: $x^2$ is **not** a quadratic factor–it is a repeated linear, so treat it as such.
  - Common mistake 2: You only need to put $Ax + B$ in the numerator for quadratic factors.

6. Improper Integrals

- **When to use:** When the integral involves $\infty$ or when the stuff you're integrating isn't defined somewhere in the interval you're integrating over.
- **How to use:** Four cases:
  (a) $\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$ ($-\infty$ is similar)
  (b) $\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^a f(x) + \int_a^\infty f(x)$ (then use case (a) on both)
  (c) f not defined at b: $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t$ (f not defined at a is similar)
  (d) f not defined at c(between a and b): $\int_a^b f(x) = \lim_{t\to c^-} \int_a^t f(x) + \lim_{s\to c^+} \int_c^b f(x)$

  Comparison test: If it's always bigger than something that diverges, then it diverges. If its always less than something that converges, then it converges.
- **Reminders, Tips, and Tricks:**
  - If it's case (a) or (d), then we say the integral converges only if **both** of the limits converge.
  - Don't forget to write whether its a limit from the left or from the right!
  - For the comparison test, pick something simple to comare it to: usually $\frac{1}{x^p}$ or something similar.
  - $\int_1^\infty \frac{1}{x^p}dx$ converges if and only if $p > 1$; $\int_0^1 \frac{1}{x^p}dx$ converges if and only if $p < 1$

7. Approximate Integration:

- **When to use:** When the problem tells you to...
- **How to use:** Memorize/write down the formulas (and their Error bounds!) for each of the rules.
- **Reminders, Tips, and Tricks:**
  - The first step in determining error bounds is to find a bound on $|f''|$ (for Trapezoid and Midpoint) or $|f^{(4)}|$ (for Simpson's). One strategy for doing this is to use the triangle inequality ($|x \pm y| \leq |x| + |y|$) and try to "throw away" trig functions, denominators, etc.
  - Simpson's rule only works for even $n$.
  - Make sure the direction of your inequality signs makes sense.

8. Arclength/Surface Area Of Revolution: Not much to say here... Just remember the formulas:

- Arclength: $\int_a^b \sqrt{1 + (f'(x))^2}dx$
- Surface area: $2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}dx$