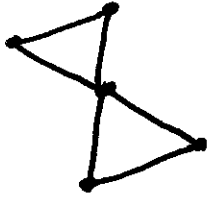


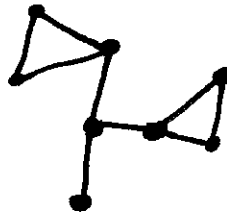
Connectivity, Cuts

1. Find the cut edges and cut vertices in each of the following graphs.

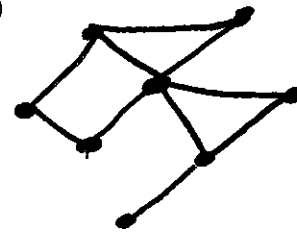
(a)



(b)



(c)



2. Show that if G is a simple graph on n vertices with a connected component of k vertices, then it has at most $\binom{k}{2} + \binom{n-k}{2}$ edges.

Note: this will be helpful for # 43 from 9.4

3. An Euler path is a simple path that uses every edge. Prove that a connected simple graph in which all vertices have even degree has an Euler path.

4. Prove that a vertex incident to a cut edge is a cut vertex iff it is not pendant.

5. A farmer needs to get a chicken, a fox, and a bag of corn across a river in a small boat that is only large enough for him and one piece of cargo. However, the chicken and the corn cannot be left alone together, nor can the fox and the chicken since otherwise one would get eaten.

(a) Draw a graph modeling this problem. In particular, you should have a vertex for each of the allowable states (who's on which side of the river), and an edge between two states if the farmer can make one trip across the river to move between those states.

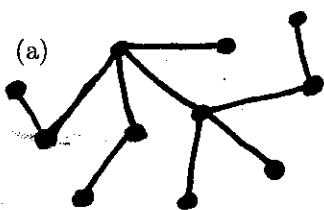
(b) Find a solution to this puzzle by finding a path through your graph from the vertex that represents all 4 on one side of the river to the vertex that represents all 4 on the other side.

6. (Tricky) Prove that a simple graph is bipartite iff it has no cycles of odd length.

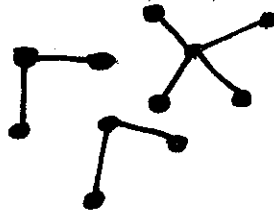
Trees

1. Determine whether each of the following are trees and, if so, if it is a full m -ary tree for some m .

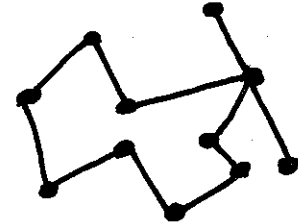
(a)



(b)



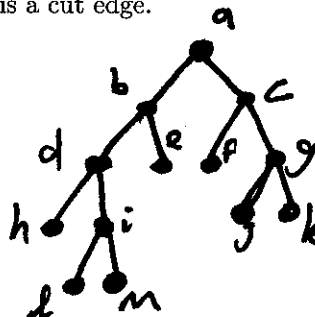
(c)



2. Prove that a simple connected graph G is a tree iff every edge is a cut edge.

3. Consider the rooted tree pictured at right.

- (a) What is the root?
- (b) What are the leaf nodes?
- (c) Draw the subtree rooted at e
- (d) Is this a full m -ary tree for any m ?
- (e) What are the descendants of d ?



4. How many different non-isomorphic trees are there with 4 vertices? What about rooted trees?

5. Prove that every tree is bipartite. Are all bipartite graphs trees?