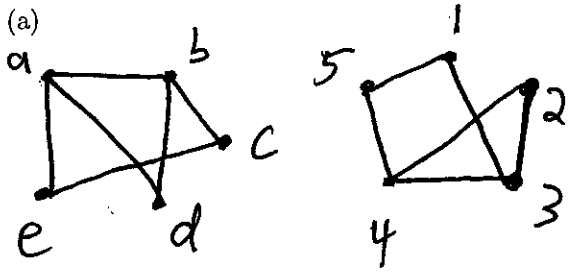
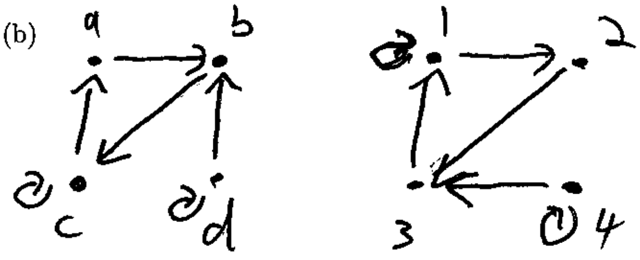
Isomorphisms

1. Give an explicit isomorphism between each of the following pairs of graphs:

(a)

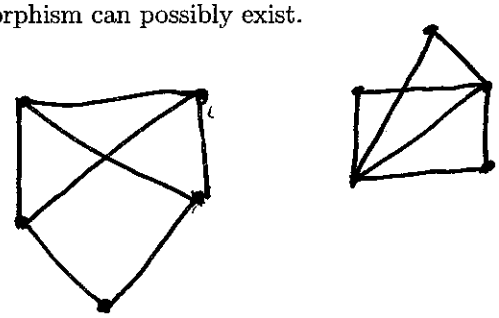


(b)

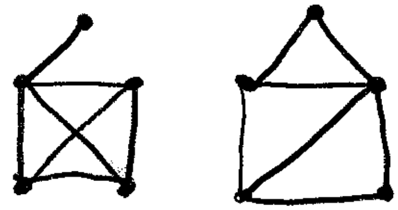


2. Show that the following pairs of graphs are **not** isomorphic. You should be sure to explain why no isomorphism can possibly exist.

(a)



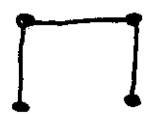
(b)



3. How many non-isomorphic graphs are there with three nodes? With four?

4. For any graph G, the complement of G, denoted $\overline{G}$, is the graph with the same vertex set as G and an edge between u and v iff G does not have an edge between u and v

   (a) What is the complement of C_4?

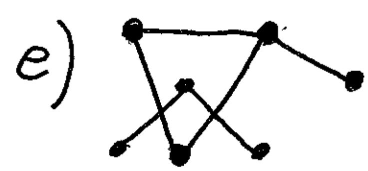
   (b) A graph is called "self-complementary" if $G \cong \overline{G}$. Show that  is self-complementary.

   (c) Show that if G is a self-complementary graph with n vertices, then $n \equiv 0$ or $1 \pmod 4$. Hint: how many edges are in a self-complimentary graph with n vertices?

   (d) (Tricky–finish the rest and come back to this) Prove the following partial converse of part (c): If $n \equiv 0$ or $1 \pmod 4$, then there is a self-complimentary graph with n vertices. Hint: induction.

5. Find examples of non-isomorphic graphs G_1, G_2 such that both G_1 and G_2 have the same number of nodes with each given degree. Note that this shows that checking the degrees is not enough to determine whether two graphs are isomorphic

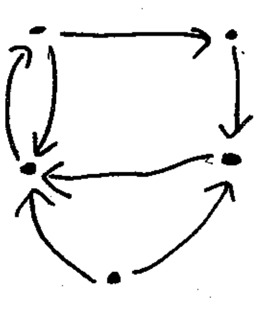
6. True/False? If $G_1 \cong G_2$ and $H_1 \cong H_2$, then $G_1 \cup H_1 \cong G_2 \cup H_2$.



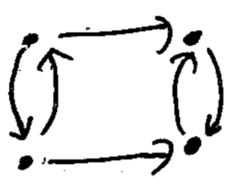
Connectivity

1. Find the (strongly) connected componenets in each of the following graphs.

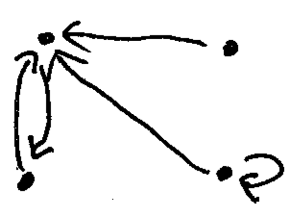
(a)



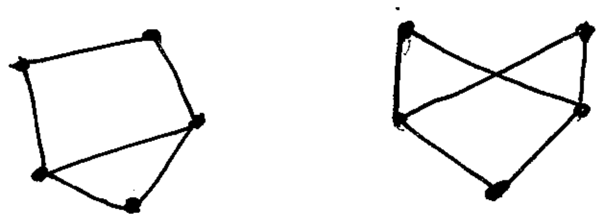
(b)



(c)



2. Use a path/connectivity argument to show that the following graphs are **not** isomorphic



3. Show that the relation "there is a path from a to b and from b to a" is an equivalence relation on the set of nodes of a directed graph. What are the equivalence classes?

4. Why do we not talk about "weakly connected componenets"?

5. An Euler path is a simple path that uses every edge. Prove that a connected simple graph in which all vertices have even degree has an Euler path.

6. (Tricky) Prove that a simple graph is bipartite iff it has no cycles of odd length.