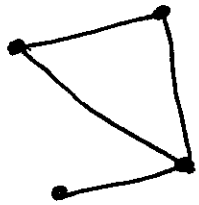
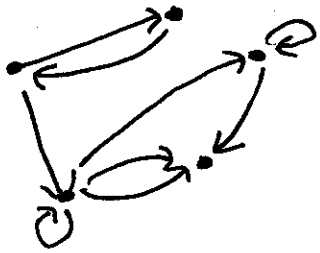
Graphs

1. Decide whether each of the following graphs are simple graphs, multigraphs, directed simple graphs, or directed multigraphs.

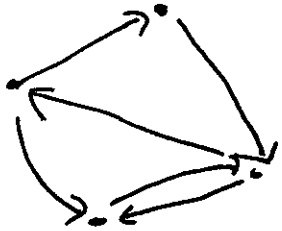
(a)



(b)



(c)



2. For each of the graphs in problem 1, find the degree (or in-degree and out-degree for directed graphs) of each vertex, and identify the isolated and pendant nodes. Verify also that the sum of the degrees is always twice the number of edges.

3. How many edges are in in each of the following graphs?

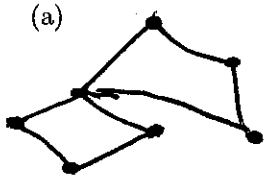
   (a) C_4
   (b) K_6
   (c) C_n
   (d) K_n
   (e) $K_{m,n}$

4. Is there a graph with 25 vertices, each of degree 3?

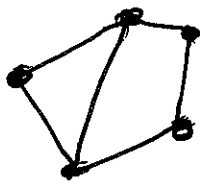
5. Find a graph with 10 vertices, each with degree 4.

6. Which of the following graphs are bipartite?

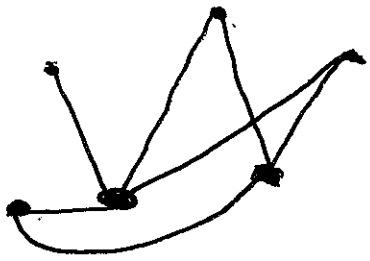
(a)



(b)



(c)



7. For which values of n are each of the following graphs bipartite?

   (a) K_n
   (b) C_n
   (c) Q_n (the n-cube)

8. True/False. For those that are true, prove it. For those that are false, provide a counterexample.

   (a) The union of two bipartite graphs is bipartite
   (b) The intersection of two bipartite graphs is bipartite

9. Find all subgraphs of K_3

10. Suppose G is a simple graph in which all vertices have even degree. Prove that for every non-isolated vertex, you can find a path from that vertex back to itself that never uses any individual edge twice.