

Independence, Conditional Probability

1. Suppose you randomly generate a bit string of length 9 and for each position, you pick a 1 with probability $\frac{1}{3}$ and a 0 with probability $\frac{2}{3}$. What is the probability that you get at least one 1?
2. Determine whether each of the following events are independent where the sample space is the set of all possible outcomes when tossing a fair coin twice
 - (a) (The First Coin is Heads) and (The Second is Tails)
 - (b) (The First is Heads) and (The two are the same)
 - (c) (The First is Heads) and (Both are heads)
3. Repeat the above problem, but for a coin that comes up heads with probability $p \neq \frac{1}{2}$.
4. Are disjoint events independent? Prove your answer.
5. Prove that if E and F are independent events, then \overline{E} and F are also independent.
6. What is the probability that you get exactly 3 heads when flipping a fair coin 8 times if the first two flips turn up tails?
7. Suppose you roll two dice and at least one comes up a 3. What is the probability that the sum of the dice is at least 7?
8. Consider the following game: two people take turns drawing balls from a bin with n red balls and 1 green ball. Balls are not returned to the bin after being drawn and the person who draws the green ball wins. If n is odd, what is the probability that the first player wins?
9. Suppose you have a big bin with 50 red balls and 30 blue balls from which you draw 3 balls without replacement. What is the probability that all three are the same color?