

**Basic Probability**

- Suppose you flip 3 fair coins.
  - What is the sample space for this setup?
  - In terms of sets, what is the event described by “at least 2 heads”?
  - What is the probability of this event?
  - Repeat (b) and (c) for the event “the coins come up the same”
- Determine the probability for each of the following events. What is the sample space in each case?
  - Getting 6 heads when you flip a fair coin 6 times?
  - Getting 4 heads when you flip a fair coin 6 times?
  - Getting a flush in poker? (a flush means all cards are the same suit)
  - Winning the lottery by picking 7 correct numbers out of 50 choices?
- Suppose you enter a raffle in which 10 prizes will be given out by drawing names at random from a hat with 100 names in it. What is the probability that you win at least one prize if:
  - Names are not replaced after being drawn?
  - Winners get put back in the hat and can potentially win again?
- Show that in Yahtzee (roll 5 dice) you are more likely to get a 2-of-a-kind than a 3-of-a-kind even though there are the same number of possible 3-of-a-kinds as 2-of-a-kinds.
- Find, with proof, the optimal strategy for the Monty Hall problem with 4 doors instead of 3.
- Show that the order of poker hands is correct, in the sense that better hands have lower probabilities. For reference, the order is:

(a) Royal Flush	(f) Straight
(b) Straight Flush (starting with A is OK)	(g) Three of a kind
(c) Four of a kind	(h) Two of a kind
(d) Full House	(i) High Card (nothing from above)
(e) Flush	

**Non-Uniform Distributions, Independence**

- What is the probability of getting 3 heads when flipping a coin 5 times if the coin is twice as likely to come up heads as tails?
- Suppose you randomly generate a bit string of length 9 and for each position, you pick a 1 with probability  $\frac{1}{3}$  and a 0 with probability  $\frac{2}{3}$ . What is the probability that you get at least one 1?
- Determine whether each of the following events are independent where the sample space is the set of all possible outcomes when tossing a fair coin twice
  - (The First Coin is Heads) and (The Second is Tails)
  - (The First is Heads) and (The two are the same)
  - (The First is Heads) and (Both are heads)
- Repeat the above problem, but for a coin that comes up heads with probability  $p \neq \frac{1}{2}$ .
- Are disjoint events independent? Prove your answer?