Stirling Numbers, Derrangements, Indistinguishable Boxes

Note: most of these problems don't have particularly nice solutions–your answers will almost surely involve either Stirling Numbers or $p_k(n)$'s

1. Find $p_4(4)$ and $p_3(5)$
2. (a) How many ways are there to distribute 6 identical books into 4 identical boxes?
   (b) What if each box must contain at least one book?
3. How many ways are there to distribute 9 balls into 5 boxes if each box must have at least one ball and:
   (a) both the balls and boxes are labeled?
   (b) the balls are all identical, but the boxes are labeled?
   (c) the balls are labeled but the boxes aren't?
   (d) neither the balls nor the boxes are labeled?
4. How many ways can you split a set of n elements into 2 disjoint subsets? Note: you should be able to write down an explicit formula for this.
5. How many onto functions are there from a set of 5 elements to a set of 3 elements?
6. How many ways can you return n hats to n people so that
   (a) Exactly one person gets their original hat back?
   (b) At least two people get their original hats back?
7. Show that $n! = \binom{n}{0}D_n + \binom{n}{1}D_{n-1} + \cdots + \binom{n}{n}D_0$
8. Show that the sequence D_n (the number of derrangements of n objects) satisfies the recurrence relation $D_n = (n-1)(D_{n-1} + D_{n-2})$
9. Find a generating function for the number of partitions of n that have only even-sized pieces.
10. (Tricky but Cute) Show that for any positive integers n, k, the number of partitions of n that have k parts is the same as the number of partitions of n that have k as their largest part.