

Inclusion-Exclusion

1. Suppose there are 200 math majors, 300 computer science majors, and 150 cog sci majors at UC Berkeley. Suppose also that for each pair of majors, there are 75 students majoring in at least those subjects, and suppose there are 5 triple majors. How many students are majoring in at least one of these three subjects?
2. How many positive integers not exceeding 1000 are coprime to 2, 3, and 5?
3. How many ways are there to rearrange the letters of the English alphabet so that CAT, DOG, and FISH are all absent.
4. Find the number of solutions in non-negative integers to $x_1 + x_2 + x_3 = 15$ where $x_1 \leq 6$, $x_2 \leq 8$, $3 \leq x_3$
5. How many prime numbers are there not exceeding 200? Hint: $\sqrt{200} \approx 14.4$
6. How many positive integers not exceeding 1000 are coprime to 84?
7. How many bit strings of length 12 either start with 00, end with 01, or start with xy101 where x, y can each be either 0 or 1?
8. Suppose you have n sets, each of size m , and you know every k of them shares a_k elements in common. Find a formula for the number of elements in $A_1 \cup A_2 \cdots A_n$ in terms of binomial coefficients and the a_k sequence.