

Indistinguishable Objects/Boxes (and some leftover problems from last time)

1. How many ways are there to rearrange the letters in the word SUCCESS?
2. The 55 students of math 55 have once again found a bag of 555 gold coins. How many different ways can they divide it among themselves if
 - (a) there are no restrictions on the distribution?
 - (b) each student must get at least one piece of gold?
 - (c) the fiercest student gets 300 pieces?
3. How many ways are there to choose 5 coins if you have an infinite supply of pennies, nickels, dimes, and quarters?
4. Suppose you have 4 nickels, 3 dimes, and a quarter. In how many different orders can you put your 75 cents into a vending machine?
5. How many solutions in non-negative integers are there to
 - (a) $x_1 + x_2 + x_3 + x_4 + x_5 = 30$
 - (b) $x_1 + x_2 + x_3 + x_4 + x_5 \leq 30$ (Hint: create a dummy variable)
 - (c) $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ and $x_1 \geq 2$
 - (d) $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ and $x_1 \leq 10$
6. How many ways can you put n books on a bookcase with r shelves?
7. How many positive integers less than 1,000,000 have the sum of their digits equal to 9?
8. In bridge, each of 4 players is dealt 13 cards from a standard 52-card deck. How many different possibilities are there for such a setup?
9. Prove that $\binom{2n}{2} = 2\binom{n}{2} + n^2$. Hint: for the RHS, break it down into cases of where the 2 elements come from.
10. Prove that $\binom{n+r+1}{r} = \binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \dots + \binom{n+r}{r}$. Hint: the RHS counts the number of ways to give r indistinguishable objects to $n+2$ people.
11. A rectangular city's roads are laid out like a grid in which $(0,0)$ is the most southwest corner of the city and (m,n) is the most northeast. If there's a road at every integer (both north-south and east-west), how many ways can you get from $(0,0)$ to (m,n) assuming you never backtrack (that is, you never go south or west)?
12. Prove that $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$ by using a combinatorial argument