

Permutations, Combinations

- How many bit strings of length n
 - contain exactly four 1's?
 - contain at least two 0's?
 - contain the same number of 0's and 1's? Assume n is even.
- Recall that a poker hand consists of 5 cards from a standard 52-card deck. How many ways are there to get a:
 - four-of-a-kind?
 - two-of-a-kind?
 - hand with no pairs?
 - flush? (All cards the same suit)
 - straight? (A straight is five cards with consecutive ranks, but can be of any suit)
 - straight flush? (combine the above two)
- How many odd-sized subsets are there of a set with 10 elements? With n ? Your answer should be a closed form.
- Show that if p is prime and $1 \leq k < p$, then $\binom{p}{k}$ is divisible by p
- How many different 5-person committees can be made if there are 100 students and 30 faculty members, and the committee must contain more students than faculty?

Binomial Theorem and Combinatorial Proofs

- Show that $\sum_{k=0}^n 2^k \binom{n}{k} = 3^n$
- What is the coefficient of x^3y^5 in the expansion of $(2x + 3y)^8$. Hint: be careful...
- Prove that $\binom{2n}{2} = 2\binom{n}{2} + n^2$. Hint: for the RHS, break it down into cases of where the 2 elements come from.
- A rectangular city's roads are laid out like a grid in which $(0,0)$ is the most southwest corner of the city and (m,n) is the most northeast. If there's a road at every integer (both north-south and east-west), how many ways can you get from $(0,0)$ to (m,n) assuming you never backtrack (that is, you never go south or west)?
- Prove that $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$ by using a combinatorial argument