Counting

1. If you have 2 different styles of shirts, each of which comes in 4 colors, 4 pairs of jeans, and 3 pairs of shoes, how many different outfits can you make?
2. How many strings of 5 decimal digits
   (a) contain at least one 4?
   (b) do not contain the same digit twice?
   (c) do not have two consecutive symbols that are the same?
   (d) either end in 4, or start with 6?
3. (a) How many different functions are there from a set with n elements to a set with m elements?
   (b) How many different injective functions are there from a set with n elements to a set with m elements? You may assume $n \leq m$
4. How many different possible truth tables are there for a proposition with n propositional variables?
5. How many positive integers ≤ 100 are divisible by either 3 or 4?
6. How many ways are there to rearrange the alphabet so that
   (a) BAD never appears?
   (b) $MATH$ does appear?
   (c) either FUN or $MATH$ (or both) appear?
7. How many palindromes of length n are there? Hint: handle n even and odd differently

Pigeon Hole Principle

1. Suppose you have 4 different colors of socks. If the room is too dark to see anything, how many socks must you take out of the drawer before you are guaranteed to have a matching pair?
2. Show that among any set of 56 integers, there is a pair whose difference is divisible by 55
3. Show that if there are 51 houses on a block, numbered between 100 and 199, then there must be two houses with consecutive addresses.
4. Show that if you select 28 integers from the first 54 positive integers, there is a pair whose sum is 55.
5. Show that if you select 5 points in the xy-plane with integer coordinates, there is at least one pair whose midpoint has integer coordinates

Induction-based Puzzles

1. The 55 students of math 55 have found a large bag of gold containing 555 gold pieces. In order to divide it up, they decide to use the following system: at the first stage, the fiercest student proposes a distribution of the gold pieces and all students (including the fiercest) vote on it. If at least half (that is, 55/2) vote in favor, they use that distribution. Otherwise, the student is permanently banned from Evans Hall and the next fiercest student proposes a new distribution which now requires at least half of the remaining students' (ie, 54/2) votes to pass, etc, etc. You may assume students all prefer more gold to less, prefer not to be banished from Evans, and all else being equal like to kick other people out. How much gold does the fiercest student get? Hint: It's a lot.
2. Suppose you host a Ping-Pong tournament with n people in which everyone plays everyone else (that is, each person plays $n-1$ games). Show that no matter what the outcomes of the matches, you can put all the participants in a line such that for all i the ith person in line beat the $i+1$st person in line.