Homogeneous Linear Recurrence Relations with Constant Coefficients

1. Find the general solution to each of the following recurrence relations:
   (a) $a_{n+2} = 2a_{n+1} + 3a_n$
   (b) $a_n = 6a_{n-1} - 9a_{n-2}$
   (c) $a_n = 4a_{n-1}$
   (d) $a_n = 6a_{n-1} - 8a_{n-2}$
2. Find the solution to the recurrence relation $a_{n+2} = 5a_{n+1} - 6a_n;\ a_0 = 3, a_1 = 8$
3. Find the solution to the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2};\ a_0 = -2, a_1 = 6$
4. A theorem from analysis says that r is a double-root of $r^2 + br + c$ if and only if r is also a root of the derivative, $2r + b$. Use this fact to show that if r is a double root of $r^2 + br + c$, then nr^n is a solution to the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$

Non-homogeneous Recurrence Relations

1. Determine whether each of the following are homogeneous or non-homogeneous.
   (a) $a_n = 3a_{n-1} + 2a_{n-2} + n^2$
   (b) $a_n = a_{n-1} - 4a_{n-2} + 1$
   (c) $a_n = a_{n-1} + 7a_{n-2} + a_{n-3}$
2. What is the "form" of your trial solution for each of the following. For convenience, the roots (and their multiplicities) of the associated homogeneous equation are given:
   (a) $a_n = -a_{n-1} + 6a_{n-2} + 2n;\ (r = 2, -3)$
   (b) $a_n = 4a_{n-1} - 4a_{n-2} + n2^n;\ (r = 2, 2)$
   (c) $a_n = 3a_{n-1} - 2a_{n-2} + n^3;\ (r = 1, 2)$
3. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + n;\ a_0 = -\frac{3}{4}, a_1 = -\frac{5}{4}$
4. Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 2^n;\ a_0 = \frac{13}{3}, a_1 = \frac{44}{3}$

Induction-based Puzzles

1. The 55 students of math 55 have found a large bag of gold containing 555 gold pieces. In order to divide it up, they decide to use the following system: at the first stage, the fiercest student proposes a distribution of the gold pieces and all students (including the fiercest) vote on it. If at least half (that is, 55/2) vote in favor, they use that distribution. Otherwise, the student is permanently banned from Evans Hall and the next fiercest student proposes a new distribution which now requires at least half of the remaining students' (ie, 54/2) votes to pass, etc, etc. You may assume students all prefer more gold to less, prefer not to be banished from Evans, and all else being equal like to kick other people out. How much gold does the fiercest student get? Hint: It's a lot.
2. Suppose you host a Ping-Pong tournament with n people in which everyone plays everyone else (that is, each person plays $n - 1$ games). Show that no matter what the outcomes of the matches, you can put all the participants in a line such that for all i the ith person in line beat the $i + 1$st person in line.