**Strong Induction**

1. Prove that if $n \geq 18$, then you can make $n$ cents out of just 4- and 7-cent stamps
2. Consider a game in which two players alternate turns taking as many stones as they want from one of two piles of stones. The player who removes the last stone wins.
   (a) Show that if the two piles start with the same number of stones, then the second player can always win.
   (b) Show that even if the piles start with different amounts, the second player can still always win.
3. Here we'll use well-ordering to show that $x^2 + y^2 = 3xyz$ has no solutions in positive integers.
   (a) Show that any solution must have $x \equiv 0 \pmod 3, y \equiv 0 \pmod 3$.
   (b) Use well-ordering to show that if there are any solutions, there must be at least one that makes $x + y + z$ minimal. Call it $(x_0, y_0, z_0)$.
   (c) Combine parts $a, b$ to show that $(\frac{x_0}{3}, \frac{y_0}{3}, z_0)$ must also be a solution.
   (d) Explain why this is a contradiction.
4. Use the principle of well-ordering to show that $\gcd(a, b)$ is the least positive integer that can be written in the form $sa + tb$ Hint: division algorithm

**Structural Induction**

1. What's wrong with the following "proof" that every Fibonacci number is even?

   $F_0 = 0$ is even. If $F_n$ and $F_{n-1}$ are even, then $F_{n+1} = F_n + F_{n-1}$ is the sum of two even numbers and thus is even. Thus, by induction, all fibonacci numbers are even.
2. Consider the set $S$ defined recursively as follows: $0 \in S, 1 \in S, \lambda \in S$ and whenever $w \in S$, both $1w1$ and $0w0 \in S$. What is simple definition for this set?
3. Prove that in any bit string, 01 occurs at most one more time that 10.
4. Consider the set $S$ defined by: $3 \in S$ and whenever $x, y \in S, x + y \in S$ and $x - y \in S$. Show that every element of $S$ is divisible by 3.
5. Prove that in any well-formed formula, the parentheses are "balanced" in the following sense: If we start from 0 and scan the formula from left to right, adding 1 every time we see a "(" and subtracting 1 everytime we see a ")," then our running total will always be non-negative at every step.

**Induction-based Puzzles**

1. The 55 students of math 55 have found a large bag of gold containing 555 gold pieces. In order to divide it up, they decide to use the following system: at the first stage, the fiercest student proposes a distribution of the gold pieces and all students (including the fiercest) vote on it. If at least half (that is, 55/2) vote in favor, they use that distribution. Otherwise, the student is permanently banned from Evans Hall and the next fiercest student proposes a new distribution which now requires at least half of the remaining students' (ie, 54/2) votes to pass, etc, etc. You may assume students all prefer more gold to less, prefer not to be banished from Evans, and all else being equal like to kick other people out. How much gold does the fiercest student get? Hint: It's a lot.
2. Suppose you host a Ping-Pong tournament with $n$ people in which everyone plays everyone else (that is, each person plays $n - 1$ games). Show that no matter what the outcomes of the matches, you can put all the participants in a line such that for all $i$ the $ith$ person in line beat the $i + 1$st person in line.