

Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

Linear Congruences and the CRT

1. Find an inverse of 9 mod 20.

2. Solve the congruence $6x \equiv 15 \pmod{21}$

3. Solve the system of congruences
$$\begin{cases} x \equiv 3 \pmod{4} \\ x \equiv 2 \pmod{5} \\ x \equiv 5 \pmod{7} \end{cases}$$

4. Suppose we want to find all solutions to $x^2 \equiv 4 \pmod{35}$

(a) Explain why any such solution must also be a solution to the system $\begin{cases} x^2 \equiv 4 \pmod{5} \\ x^2 \equiv 4 \pmod{7} \end{cases}$ and vice-versa.

(b) Solve each of the congruences in (a) individually to show that $x \equiv 2$ or $3 \pmod{5}$ and that $x \equiv 2$ or $5 \pmod{7}$

(c) Use the CRT on each of the possibilities from (b) to find all solutions to $x^2 \equiv 4 \pmod{35}$

5. Compute $7^{130} \pmod{11}$. Hint: this requires 1-2 lines of work.