Math 55 Worksheet

Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

Primes and GCD

1. Determine the GCD of each of the following pairs of numbers. Determine whether each pair is relatively prime.

   (a) 36, 75

   (b) 539, 75

   (c) 55, 70

   (d) 16, 120

   (e) $2^2 3^4 5^3, 3 5^2 7^6$

   (f) $3^4 5^2 7^2, 3^3 7^2 11$

2. Is the set of integers $\{3^2 5^4, 2^2 7^3, 11^5 13^3\}$ pairwise relatively prime?

3. The function $\phi : \mathbb{N} \to \mathbb{N}$ defined by $\phi(n) = \#$ of positive integers less than n that are relatively prime to n.

   (a) Find $\phi(12), \phi(11), \phi(25), \phi(121)$

   (b) Prove that n is prime iff $\phi(n) = n - 1$

   (c) Show that if p is prime, then $\phi(p^2) = p^2 - p$

4. (a) Find all solutions in positive integers to $x^2 = 11 + y^2$

   (b) Find all solutions in positive integers to $x^2 = 6 + y^2$

5. Prove that for any positive integers a and b, $ab = \gcd(a, b) \cdot \operatorname{lcm}(a, b)$. Hint: think about prime factorizations.

6. Find all prime numbers p such that $p^3 + 3^p$ is also prime.

7. Prove that a positive integer n is a perfect squre iff every exponent in its prime factorization is even.

8. (Tricky) Prove that if $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$, then $a \equiv b \pmod{\operatorname{lcm}(m, n)}$.
   Hint: recall that if $m|x$ and $n|x$, then $\operatorname{lcm}(m, n)|x$

Different Bases

1. Convert each of the following decimal numbers to the indicated base:

   (a) 1358 to binary

   (b) 936 to octal

   (c) 474 to hexadecimal

2. Convert each of the following to ordinary decimal notation

   (a) $(1101)_2$

   (b) $(347)_8$

   (c) $(BEEF)_{16}$