Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

Divisibility and Modular Arithmetic

1. Evaluate each of the following:
   (a) $-17 \mod 2$
   (b) $144 \mod 7$
   (c) $199 \mod 19$
   (d) -101 **div** 13
2. What is $\underbrace{111\cdots1}_{1000\ 1's} \mod \underbrace{11111111}_{8\ 1's}$?
3. Prove that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$
4. Show that if $n|m$ and $a \equiv b \pmod{m}$, then $a \equiv b \pmod{n}$
5. Prove that if the last digit of n is 3, then n is not a perfect square.
6. Give an example of integers a, k, l, m such that $k \equiv l \pmod{m}$, but $a^k \not\equiv a^l \pmod{m}$
7. (a) Find a solution to $5x \equiv 1 \pmod{6}$. Your answer is called a "multiplicative inverse of 5, mod 6" because it behaves similar to $\frac{1}{5}$.
   (b) Show that 2 has no multiplicative inverse mod 6. That is, show that $\frac{1}{2}$ has no meaning when working mod 6.
8. Show that a natural number n is divisible by 11 iff the alternating sum of its digits is too (ie, first digit - second + third - fourth + $\cdots$)