

**Instructions**

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

**Cardinality and Countability**

1. Show that the union of two countable sets is countable.
2. Show that the union of countably many countable sets is itself countable. Hint: don't try to write down an explicit bijection. Instead, show there is a natural way to enumerate all the elements in the union.
3. (a) Show that the set of all real numbers that are roots of polynomials with integer coefficients is countable.  
(b) Show the same thing, but for polys with rational coefficients. (FYI, such reals are called "algebraic")  
(c) Use part (b) to show that there are uncountably many real numbers that are not algebraic. (FYI, such reals are called "transcendental")
4. (a) Prove that there are a countable number of valid English language sentences  
(b) Use part (a) to show that there is a Real number that cannot be described in English  
(c) By modifying parts (a) and (b), show that there is a Real number that cannot be described in **any** language
5. The Hotel Infinity is rather unique, in that it has infinitely many rooms, one for each natural number. However, due to a recent surge in demand, all the rooms are currently full and people are still coming at a ludicrous rate. Fortunately, the owner of the place has an intercom that lets him talk to all the current occupants, and all of them are perfectly happy to switch rooms if asked.
  - (a) Given this situation, how can the owner find space for one more person?
  - (b) If a bus with infinitely many passengers (one for each natural number) shows up, how can the owner accomodate them?
  - (c) If infinitely many buses each with infinitely many passengers shows up (again, infinite=one for each natural number), how can they be accomodated?