Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

Functions and Sets

1. Determine whether each of the following are injections, surjections, bijections, or none of the three.
   (a) $f : \mathbb{R} \to \mathbb{R}, f(x) = 3x - 2$
   (b) $f : \mathbb{R} \to \mathbb{Z}, f(x) = \lfloor x \rfloor$
   (c) $f : \mathbb{N} \to \mathbb{N}, f(n) = n + 1$
   (d) $f : \mathbb{R} \to \mathbb{R}, f(x) = x(x - 3)(x + 2)$
2. Decide whether each of the following are true or false. For those that are true, prove it. For those that are false, provide a counterexample.
   (a) If f, g are injective, then $f \circ g$ is
   (b) If f, g are surjective, then $f \circ g$ is
   (c) If $f \circ g$ is injective, then f is.
   (d) If $f \circ g$ is injective, then g is
   (e) If g is not surjective, then $f \circ g$ is not surjective.
3. Prove that $(A - B) \cup C \subseteq A \cup C$
4. Prove that $A \subseteq \overline{B - A}$
5. Explain why non-injective functions have no inverse function
6. If $C \subseteq A$ and $f : A \to B$, then the restriction of f to C, denoted $f|_C$ is the function with domain C, codomain B and the property that $f_C(c) = f(c)$ for all $c \in C$.
   (a) Show that if f is injective, then $f|_C$ is injective for any $C \subseteq dom(A)$
   (b) Is the same true about surjectivity?
7. Consider the function $f : \mathbb{N} \to \{0, 1, 2\}$ defined by $f(n) =$ remainder when n is divided by 3.
8. Even when a function $f : A \to B$ is not invertible, we will often talked about the inverse image of a set $C \subseteq B$, which we denote by $f^{-1}(C)$ and is defined as $\{x | f(x) \in C\}$. Determine each of the following:
   (a) $f^{-1}(\{June29\})$ where f :(People) $\to$ (days of year), $f(x) = x's$ birthday.
   (b) $f^{-1}(\{x | x > 3\})$ where $f : \mathbb{R} \to \mathbb{R};\ f(x) = x^2$
   (c) $f^{-1}(\{3\})$ where $f : \mathcal{P}(\mathbb{N}) \to \mathbb{N}$, $f(S) = \min(S)$ (or 0 if $S = \emptyset$)
9. Using the same notation as above, prove that $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B)$ and that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$. Is the same true about images of sets (ie, $f(A), f(B)$, etc)?